# MATHEMATICS in MEDIEVAL EUROPE

Greek Mathematics → Hindu Mathematics

Arab. Mathematics

Romans
(a primitive $N^o$ system & a few facts of arithmetic & little more)

6 ↕ 12

So called "dark ages"

Scholarship in a stagnant formalized state (as indeed was most of society) and intimately attatched to the church.

12 ↕ beginning 15

For want of a better term
middle ages

Growth of trade (with arabs), manifacture & merchant activities. Gradual growth of an artisan/technician class closely associated with scholars, both being supported by wealthy "princes"

15 ↕ 17

Renaissance

out of which emerged:

modern notation ← Descartes → Proj Geom
including the function concept. Libnitz Euler
Coordinate Geometry ← Fermat
The Calculus

From the outset we must be cautious to distinguish between incidental and isolated discoveries (~~though~~ in hind-sight they may seem quite profound) which did not gain general acceptance or contribute to the general advance of knowledge and those which were adopted and passed on to latter scholars. (Perhaps due to inherently poor communications, often agravated by political/social factors, many seemingly significant ideas apparently went unnoticed, only to be re-discoverd latter.

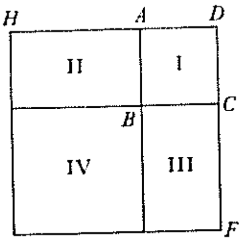


Figure 9.1

$ax^2 = bx + c$, so that $a$, $b$, and $c$ are always positive. This avoids negative numbers standing alone and the subtraction of quantities that may be larger than the minuend. In this practice of using the separate forms, al-Khowârizmî follows Diophantus. Al-Khowârizmî recognizes that there can be two roots of quadratics, but gives only the real positive roots, which can be irrational. Some writers give both positive and negative roots.

One example of a quadratic treated by al-Khowârizmî reads as follows: "A square and ten of its roots are equal to nine and thirty dirhems, that is you add ten roots to one square, the sum is equal to nine and thirty." He gives the solution thus: "Take half the number of roots, that is, in this case five, then multiply this by itself and the result is five and twenty. Add this to the nine and thirty, which gives sixty-four; take the square root, or eight, and subtract from it half the number of roots, namely five, and there remains three. This is the root." The solution is exactly what the process of completing the square calls for.

Though the Arabs gave algebraic solutions of quadratic equations, they explained or justified their processes geometrically. Undoubtedly they were influenced by the Greek reliance upon geometrical algebra; while they arithmetized the processes, they must have believed that the proof had to be made geometrically. Thus to solve the equation, which is $x^2 + 10x = 39$, al-Khowârizmî gives the following geometrical method. Let $AB$ (Fig. 9.1) represent the value of the unknown $x$. Construct the square $ABCD$. Produce $DA$ to $H$ and $DC$ to $F$ so that $AH = CF = 5$, which is one-half of the coefficient of $x$. Complete the square on $DH$ and $DF$. Then the areas I, II, and III are $x^2$, $5x$, and $5x$, respectively. The sum of these is the left side of the equation. To both sides we now add area IV, which is 25. Hence the entire square is $39 + 25$ or 64 and its side must be 8. Then $AB$ or $AD$ is $8 - 5$ or 3. This is the value of $x$. The geometric argument rests on Proposition 4 of Book II of the *Elements*.

The Arabs solved some cubics algebraically and gave a geometrical explanation in the manner just illustrated for quadratics. This was done, for example, by Tâbit ibn Qorra (836–901), a pagan of Bagdad, who was also a physician, philosopher, and astronomer, and by the Egyptian al-Hasan ibn al-Haitham, known generally as Alhazen (*c.* 965–1039). As for the general

## dark ages

all learning in latin – poor translations into latin (without proof) of some greek mathematics eg Boethius & Nichomachus.
Clearer sources of instruction were provided by the writings of Gerbert (Pope Sylvester II) in 10 though nothing new was introduced.
None-the-less what little mathematics there was, rated highly in their teaching.

- Curriculum
  - quadrivium
    - arithmetic (whole N°s & a few "named" fractions eg half; 4 ops. +, x, ÷, – all calc. being done on forms of the abac
    - Music (application of N°s in the Pythagorean Traditio
    - Geometry (study of magnitudes a rest)
    - astronomy (study of magnitude in motion)
  - trivium
    - rhetoric – persuasive speaking/writin
    - dialectic – logical disputation to investigate trut
    - Grammar – relation between words as used in writing or speech.

Fresh acceptance of the ideas of infinity & infinitesimals God/Me

# Middle Ages

Travellers (Eg Fibonacci) & Christian conquests led to the gradual entry of preserved Greek & new Arabian Mathematics. Thus Arabic numerals, the Hindu mode of calculation including the use of fractions, $\sqrt{\ }$ & $\sqrt[3]{\ }$ was eventually adopted.

Understanding of this newly discovered learning gradually increased & dominated European thinking though little new was added. Algebra was still done with words, though the use of abbreviations eventually led to a clumsy undeveloped form of symbolism. Indeed, with few exceptions eg Bacon

(13) this inpouring of knowledge inspired adoration & inhibited natural enquiry.

None-the-less, though the phenomenon being explained were often themselves inncorrect, there was an upsurge of Rational Explanation, which coupled with the growing practical knowledge of the artisan class & some pressing practical problems (surveying due to more ambitious building etc, projectile motion, out of the need to design cannon proof fortresses) led to a new "Natural philosophy" based on the belief that mathematics was the language of nature (God). Indeed these developments were more significant to latter mathematics than the mathematics of the time.

Knowledge of statics, levers & optics regained

attempts made to understand projectile motion:
→ fractional powers & geometric figures to represent rates & d…

Oresme & Buridan (1300-1360) broke with Aristotle → to introduce impetus, which when imparted to an object would maintain the motion in the absence of external resistance & which was gradually added to a falling body by natural gravity. (took as defn amnt of matter x velocity)

Since celestial spheres suffer no resistance to their motion God gives initial impetus & so they continue!

the notion of g…

Aristotle: only one force can act at any instant, when greater is spent, then lesser spends itself

Nemorarius: resolved forces into 2 components for a horizontally projected body.

first to use symbols purposefully.

Developments in Science: Motivated largely by

Motion — Navigation (tables etc.)

Projectile — astronomical

While the New Science based itself on "experiment"[4] these we usually seen as subordinate (even by Galileo) being need to confirm or suggest ~~the~~ law(s) (which could usually be reasoned from other grounds also) expressed mathematically which was then the basis for a chain of deductions or theorems.

## Renaissance

Specific Mathematical discoveries during this period were scantly ~~and the~~ were rapidly superseded by the ~~growth~~ of Mathematics of the late 17 & 18. None-the-less it was a preparation for this rapid flowering, perhaps the most important contribution being a gradual improvement in notation. However it was the successful application of existent mathematics to the newly developing physical sciences and the attendant new view points & new problems which these posed ~~that~~ set the stage for 18 development.

### Some developments

more complicated irrational numbers eg $\sqrt[m]{a+\sqrt[n]{b}}$ (Stifel 1486–?) were considered & treated in the trade of the Hindu's. Negative numbers though known and occasionally used were none the less not generally accepted as numbers. Complex numbers were also blundered into and treated formally as numbers (Eg. Cardan) though they were not accepted as "real" numbers.

Decimal notation (out of need for compact tables) was also introduced by Vieta et al & strongly advocated by

Stevin: 5.912    5⓪9①1②2③

Continued fractions & inf products also considered (, Bombelli, Wallis etc.)

logarithms: Stifel

$$\begin{array}{lcccc} G.P. & 1 & r & r^2 & r^3 \text{ etc} \\ & \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ A.P. & 0 & 1 & 2 & 3 \text{ etc.} \end{array}$$

Napier / Briggs & independently Bürgi

Tartaglia (Cardan): solution of cubic eqns. all coeffs +ve so many sep. cases considered.

### Symbolism/Notation

15  + ≡ p , m ≡ − by 16 + & − in use (from German merchants

= Recorde (1510–58)

> < Hariot

parentheses from about 1580. (√ not till Descartes)

Eg Cardan:

$\sqrt{7+\sqrt{14}}$ R V 7 p R 14

all that follows

$a \times b = c$ as

a
b
c

a symbol for the unknown, though used by Diophantus was slow to be introduced

Cardan (1501 - 1576)

$x^2 = 4x + 32$ --- qdratu aeqtur 4 rebus p : 32

thing

"

Coss in German
hence algebra the
"Cossic art".

Once symbols were addoped different ones were used for the various powers

Eg $x^3 + x^2 + x \equiv$ C p Z p R

cubus zenus (res = thing)

gradually exponents used.

Eg Chuquet (1484) $8^3, 7^{1m}$ $\equiv 8x^3 \cdot 7x^{-1}$

Bombelli (1526 - 1573)

$1 + 3x + 6x^2 + x^3 \equiv 1 \text{ p } 3^{\smile 1} \text{ p } 6^{\smile 2} \text{ p } 1^{\smile 3}$

Stevin (1548 - 1620) $\equiv 1^{⓪} + 3^{①} + 6^{②} + ③$

also used ½ ⅓ for
root & cube root. etc.

this → Descartes

$1 + 3x \cancel{+} 6xx + x^3$

(and sometimes $x^2$)

adopted by Gauss.

Vieta also used [illegible]

## The Growth of Symbolism and Notation in Renaissance Europe

In the 15th century the use of p and m as abbreviations for + and − was common.

By the 16th century + and − were in use, having been adopted from German merchants.

= introduced by Recorde (1510-1558).

>,< by Harriot (1560-1621).

Parentheses from about 1560 onwards.

$\sqrt{\ }$ not till Descartes, 1596-1650.

Previously ℞ had been used.

<u>Eg</u>. Cardan (1501-1576) wrote $\sqrt{7+\sqrt{14}}$ as ℞V7 p ℞14, if A, B, C were such expressions he would set out A×B = C as:

A<br>
B<br>
est C.

Decimals, successfully introduced by Vieta and strongly advocated by Stevin (1548-1620): 5.912 written as 5⓪ 9① 1② 2③.

Although used by Diophantus (Greek, ~ 250 A.D.) a symbol for the unknown was slow to be introduced. Initially it was an abbreviation for "thing" (R from res in latin; c from coss in German, hence "Cossic Art".) Initially different symbols used for various powers.

<u>Eg</u> $x^3 + x^2 + x$ as C p Z p R

cubus zenus res

Chuquet (1484) introduced exponents: Writing

$8x^3 \cdot 7x - 1$ as $8^3,\ 7^{1m}$.

However Cardan still wrote

$x^2 - 4x + 32$ as qdratu aeqtur 4 rebus p: 32

thing

Bombelli (1526-1573), following Chuquet wrote

$1 + 3x + 6x^2 + x^3$ as $1\ p\ 3^{\overset{1}{\smile}}\ p\ 6^{\overset{2}{\smile}}\ p\ 1^{\overset{3}{\smile}}$

and Stevin (1548-1620) used 1⓪ + 3① + 6② + ③; he also used fractional exponents denoted by (½), (¼) etc.

Viéte (Vieta) (1540-1603) was probably the first to use symbols purposely. He introduced x, y, z for unknowns and also allowed variable coefficients denoted by a, b, c etc.

Thus by the time of Descartes it was "natural" to write

$1 + 3x + 6xx + x^3$

(and sometimes $x^2$, though this was not fully adopted till Gauss).