Referee Report on the manuscript
"The Douglas-Rachford algorithm in the absence of convexity"
by Jonathan M. Borwein and Brailey Sims

The manuscript contains interesting mathematics on the Douglas-Rachford algorithm, especially in the nonconvex case which so far lacks good analyses. The manuscript should be revised taking the following points into account.

1. Please submit the next version with references compiled. It was hard to read this manuscript with "[?]" all over the place.

2. Please spell check the manuscript before submission. Some typos found include:

   (a) Page 4, middle: "the the".

   (b) Page 15, Remark 4: "waek topology".

   (c) Page 16: "Douglas-Ratchford".

   (d) Page 19, Reference [11]: "Spltting".

3. Page 2, middle: It would be more clear to write "$L := \alpha b + \mathbb{R}a$" as it is not clear what in the current definition of L the free variables are.

4. Page 3, first displayed equation for $T_{S,L}$: Delete "h".

5. Page 5, top: The product "Π" notation in the definition of A and $\tilde{X}$ is not optimal without explaining what the order in the product is. It is less ambiguous to write $A_1 \times \cdots \times A_M$ etc. You do this for B and P_B anyway. Also, the subscript "2" in the definition of $\tilde{X}$ is presumably meant to indicate that this is a Hilbert space? Why not simply write in line 3: "... of the Hilbert product space X^M"? $\tilde{X}$ is not used again anyway as far as I can tell. Finally, it would be more clear to write "$R_A(x) = \left(R_{A_1}(x_1), \ldots, R_{A_M}(x_M)\right)$".

6. Page 5, Example 1: The first projection (and hence all subsequent ones) has a typo: "$+$" should be "$-$".

7. Page 8, Theorem 1: Please provide an exact reference to Perron's reference (assuming it is in a book). Also, please make mathematically precise "isolated solution".

8. Page 9, to: In general, $S \cap L$ is only a *subset* of the set of fixed points of $T_{S,L}$. While in your setting it is clear that here $S \cap L$ is unique in the positive open halfspace, it is not clear to me why the set of fixed points of $T_{S,L}$ is unique there. So I don't see why you can speak of an "isolated fixed point of $T_{S,L}$" without further justification.

9. Page 9, line $+4$: Please replace ", Then" by ". Then".

10. Page 10, Theorem 2: In what sense do you use the adjective "critical" here? Presumably not in the sense of having-gradients-equal-to-zero, in which case it is more clear to delete "criticial".

11. Page 14, top, regarding the following

$$x_{n+1}(2) - 1 = x_n(2)(1 - 1/\rho_n) > 1.$$

   (a) Should the last "1" be "0"?

   (b) If so, why is $x_n(2) > 0$? I don't see this. Is it part of the assymption on the starting point, i.e., $x_0(2) > 0$. If so, Theorem 4 on that page needs to be modified accordingly.

12. Page 14, Theorem 5:

   (a) Replace "$h > 1$" by "$\alpha > 1$".

   (b) "Initial point $x_n(2)$": $x_n(2)$ is not a point to me, but rather a coordinate of one. Also, "initial" suggests $n = 0$?

   (c) In fact, I don't think you need to assume anything on $x_n(2)$.

   (d) The statement is confusing to me: "divergence at **an**(?) at least linear rate" suggests to me that the *quotient* is bigger than 1. In the proof, it becomes clear that you mean the *difference* of consecutive iterates. Why not state the inequality and remove any ambiguity?

   (e) In the proof, replace "$> \alpha - 1$ as $x_n(2) < \rho_n$" by "$\geq \alpha - 1$ as $x_n(2) \leq \rho_n$" which shows that nothing has to be assumed about $x_n(2)$ in the statement.

13. Page 15, Section 6, fourth paragraph, "all non-zero points on this line remain fixed under $T_{S,L}$": This is not correct, it is only true for all points of the form λb, where $\lambda > 0$, due to the evaluation of $\|\lambda b\|$.

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14. Page 15, paragraph before Remark 4: Please either remove this comment or make it more precise. It is hard to get something from it in the current form.

15. Page 15, Remark 4: If you claim the results in infinite-dimensions, then proofs are required. It is not entirely obvious (and if it were, then why not state and prove the results in that generality to start with)?

16. Page 17, top: The sentence "Nonetheless, all our results appropriately viewed continue to hold" is too vague — please provide the appropriate formulations. (It is clear that almost any mathematical result will allow for some "appropriate generalization" but perhaps different readers will have different thoughts on what these results would be.)

17. Page 17, second paragraph: The set defining A_0 misses a "}"; moreover, it is not clear what $\lambda_1, \ldots, \lambda_k$ are.

18. Page 17, second paragraph: It is more clear to write "$Q_{x_0} = A_0^\perp + \mathbb{R}x_0$".

19. Page 17, first and second paragraph: In the first paragraph you speak of the feasible points no longer being isolated, whereas in the second paragraph, you give two isolated point? Please clarify.

20. Page 17: I do not think that the extension of your results from a line to a general finite-dimensional affine subspace is routine. If you claim the results, please provide proofs. Otherwise, please formulate as conjectures.