Report on J. Borwein and B. Sims, "The Douglas-Rachford Algorithm in the Absence of Convexity"

The authors study the behaviour of the Douglas-Rachford algorithm for finding the intersection of two sets in the instance when one of two sets is a line, or a line segment, and the other set is a ring. For this example, the iterates of the Douglas Rachford algorithm can be written as steps in a diffence equation, leading to an analysis of the the Douglas-Rachford algorithm as a dynamical system. Specific comments follow.

1. document was not compiled enough to synchronize references, so I am not certain that these are consistent or complete.

2. Spelling errors throughout (e.g. Spiralling, waek, Spltting, Ratchford).

3. p3 l+13: It seems that an opportunity is lost to at least mention one large "pathology" of nonconvex projections by excluding the point $x = 0$ from the discussion, i.e. single-valuedness of the projectors. Perhaps a very brief mention that by excluding the origin you are ensuring that the projector and reflector corresponding to the circle/sphere is single-valued? Also, by excluding the origin from the discussion, you are excluding all initial points whose iterates pass through the origin (item below).

4. p4, Remark 1: "divide-and-concurr", though not so named by its inventor, is due to Pierra. Citation [9] should be replaced by author = G. Pierra,
   title = Eclatement de contraintes en parallèle pour la minimisation d'une forme quadratique,
   journal = Lecture Notes in Computer Science,
   publisher = Springer Verlag,
   address = New York,
   year = 1976,
   volume = 41,
   pages = 200-218
   Elser "repackaged" it in [9], but the idea is well established and common practice in the mathematics literature.

5. p5, Example 1: I do not really understand the point of this example. Averaged reflections were never under consideration.

6. p6, Fig.3: the points do not match the description – the very first move, if really a reflection across the line segment, would be in a direction northwest from the initial point with midpoint at the right endpoint of the linesegment. Similarly with the third move. It appears that the algorithm thinks it's working on a line, not a line segment.

7. p6, Fig.3: Why show alternating reflections? It is known that this does not converge even in the convex case. Averaging, or the addition of a Krasnoselski-Mann relaxation, is important for restoring firm nonexpansiveness of the fixed point mapping in the convex case.

8. p8, Theorem 1: should f be a mapping from $N \times \mathbb{R}^n \to \mathbb{R}^n$ instead of $\mathbb{R}^m$?

9. p8, Theorem 1: if you use n for the dimension of the domain and range, please use a different index for the iterate (instead of n).

10. p9, l+2, $T_y(x)$: This is the first time this notation is used, and it doesn't match $T_{S,L}$ very well. Consider an alternative notation?

11. p9, l+5: the fact that the fixed points are isolated seems to be important – at least as important as your choice of a LINE for the second set rather than a subspace. Indeed, as you point out in Remark 5, Theorem 1 only applies to operators with isolated fixed points. I would recommend proving that the fixed points of the operator under consideration are indeed isloated, if only to highlight this particular feature of the instance under investigation.

12. p9, "basis B": this is the first mention of the basis B, please define this.

13. p15, l+10, "If $\|x\| = 1$... the scheme breaks down at the first iteration.": Please be clearer what you mean by "breaks down". I agree that your description of the iterates no longer applies, but the iteration still seems well defined. In fact, in two dimensions I think it can be shown that the reflectors and hence the iterates are no longer single-valued but still the iterates of DR, now sets, display some sort of set convergence, i.e. the iterates converge to the line segment $(-1, 0) + t(2, 0)$ for $t \in [0, 1]$. This ties in to item #3 in this list.

14. p15, l-7: Can you be more specific what you mean by "various interval mapping analogues of Sharkovskii's theorem are operative"?

15. p17, Ex 2: though convexity is not essential to your results, single-valuedness of the projectors is, and this has some bearing on the basins of attraction.