THE SOAP FILM EXPERIMENT WITH A CUBE

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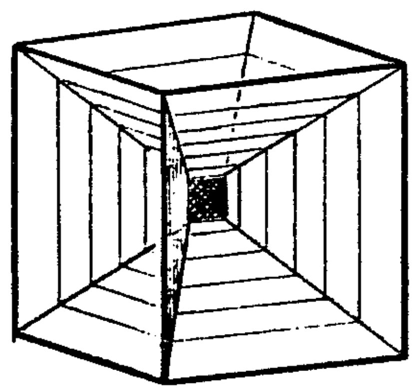
Take a wire framing a cube and lower it by a string tied on one corner into a soap film solution. When you extract the frame from the solution you have a soap film forming *13* "plane" surfaces hung between the wires in this pattern:



The property of mathematical interest in this soap film on the wire frame is that it is the surface of minimum area bounded by the wire cube frame.

The mathematical problem of finding the surface of minimum area bounded by a given closed surface in space is called Plateau's problem, after the Belgian physicist Joseph Plateau (1801 - 1883) who conducted experiments, like the soap film experiment just described, to gain insight into the mathematical solution of particular cases.

Surely, with such a pretty solution on the wire cube it would not be difficult to find the ratio of the length of the side of the interior "square" to the length of the side of the wire cube.

Let the length of the side of the wire cube be one unit and the length of the side of the interior square be x units. The surface area A is the sum of the areas of 4 congruent triangles, 8 congruent trapezia and the interior square. Using elementary trigonometry, we can calculate the area of a triangular section as $\frac{1}{2\sqrt{2}}(1-x)$ and the area of a trapezium section as $\frac{1}{2}(x+1)\sqrt{(1-x)^2+1}$. We then have an expression for A as a function of x and we want to find x so that A is a minimum.

Using the differential calculus, we find that for A to be a minimum then x is the solution of the equation:

$$A'(x) \equiv 2x - \sqrt{2} + 2\sqrt{x^2-2x+2} + \frac{2(x^2-1)}{\sqrt{x^2-2x+2}} = 0 .$$

But in this day of pocket calculators, we do not despair at complicated equations. The process of finding approximate solutions is fairly easy, especially if we have a programmable calculator. We find that $A'(0) < 0$ and $A'(.1) > 0$. Continuing, we find $A'(.072) < 0$, $A'(.073) > 0$, $A'(.0725) < 0$, and so on, to a solution $x = .07291$ correct to five decimal places. But you could hardly call this a nice solution!

Now this problem is discussed in R. Courant and H. Robbins *What is Mathematics?*, Oxford 1943 on page 387, and there it is said in an imprecise way that the soap film surfaces meet each other *"at angles of 120° along lines of intersection"*. Whichever angles this statement refers to, we cannot get our calculated solutions to produce the *120°* angle. What has gone wrong?

Our physical situation, that is, the wire cube with the soap film, has suggested a simple mathematical model: that the soap film consisted of plane surfaces. We have found x under this assumption. The actual situation is not quite this. Actually, the inside "square" is slightly bowed at the sides and the "trapezia" are not quite plane!

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SOLUTION TO "DIAMETERS"

Check with vertices of an equilateral triangle of diameter *2*. The smallest containing circle has radius $\frac{2}{\sqrt{3}}$.