Diagrammatic Reasoning in Higher Education

9 November–11 November 2018 in Newcastle, NSW

A workshop on the teaching of mathematical concepts using diagrammatic reasoning in higher education. Part of the CARMA special semester on Mathematical Thinking.

`https://carma.newcastle.edu.au/meetings/drhe/`

1 *Venue*

The workshop is held in Moot Court (room X703) in the NeW Space building of the University of Newcastle in Newcastle city centre.

Address: Hunter St & Auckland St, Newcastle

The opening reception is held at Customs House on Friday from 6–8pm.

Address: 1 Bond St, Newcastle

2 *Approximate timetable*

The reception on Friday is complimentary. Saturday and Sunday are fully catered during the workshop.

	Friday	Saturday	Sunday
9am		Dominic Verity	James Juniper
10am		Coffee	
10:30am		Judy-anne Osborn	Rafael Núñez
12pm		Lunch	
1:30pm		Richard Garner	Keith Devlin
3pm	Opening	Daniel Barter	
3:30pm	Ross Street	Coffee	
4pm		Heath Jones	
6pm	Opening Reception		

A more detailed and current timetable is available on the workshop webpage.

3 *List of abstracts*

Friday 3pm-5pm

Calculating with string diagrams

Ross Street

Motivating with linear algebra, I shall introduce symmetric monoidal categories (smc) and explain the string diagrams for which smc provide the environment. I shall show how familiar operations from vector calculus transport to smc where the properties can be expressed in terms of equalities between string diagrams. Geometrically appealing arguments will be used to prove a theorem with implications for multiplications on Euclidean space, a theorem of a type originally proved using higher powered methods.

Saturday 9am-5pm

An exploration of string diagrams in Computer Science

Dominic Verity

When I was a teenager my Computer Science teacher tortured me with *flowcharts*!

At that time, received wisdom and accepted design practice demanded that all systems should have their *logic* and *control flow* explained diagrammatically, even before a single line of code could be written. Indeed my teacher made this a condition of access to precious computing resources; I quickly learnt to hate flowcharts, abandoning them as quickly as I could.

Ultimately the advance of ubiquitous and plentiful computing resources largely eliminated flowcharts from the computing curriculum. After all why bother carefully drawing up a flowchart, as a prelude to a brief session of code entry, when one can immediately open up a dialogue with the closest machine and hack up some code? That being said these pesky flowcharts live on, in hiding, and flourish like viruses, most notably in the internals of our programming language compilers (control flow graphs), in graphical environments targeted at programming pedagogy (Blockly, Scratch, Node Red, and friends), and in the papers of theoretical computer scientists.

Had someone told me that there was interesting mathematics to be had from the world of flowcharts I might have taken these diagrams more seriously.

In this talk I will try to illustrate some of the mathematics underpinning various graphical reasoning methods in computer science. Our discussion will touch on flowcharts, finite state automata, control flow graphs, electronic circuit diagrams, and their kin. We will build upon the graphical calculus introduced in Ross Street's talk, extending it to explore structures called *traced monoidal categories*. This then will become a unifying frame for exploring the descriptive power of these diagrammatic methods and for understanding their utility in *reasoning* about and *evolving* computational systems.

Education, Math-Pictures and the Modern World

Judy-anne Osborn

I will discuss the use of diagrams and narrative in teaching mathematics. I will contrast a teaching area in which diagrams are currently used extensively: Combinatorics, with an area in which that is not so much the case: Calculus, and pose the question to what extent it needs to be this way?

Rigour and diagrams

Richard Garner

Mathematics is often considered to have a privileged epistemological status, in that true mathematical statements are not just true, but uncontroversially true. It may seem a reasonable assumption that this lack of controversy stems from the unique rigour with which mathematical arguments are invested. I will argue that diagrammatic and other non-standard reasonings provide a useful tool to interrogate this assumption, and that mathematical rigour as commonly understood is mainly a product of historical happenstance. I hope to illustrate this with examples from topology, 2-category theory and game semantics.

Diagrammatic methods for computing defect fusion in topological phases

Daniel Barter

My research is focused on (2+1)D topological quantum field theory. Recently I have been computing defect fusions in these models using diagrammatic methods. At this point, there is no other way that these computations can be done. I will explain a little bit what all this means, and demonstrate some example computations.

Diagrammatic Reasoning and the Teaching of Introductory Physics

Heath Jones

Recent decades have seen significant increases in both the number of Australian higher education providers and their student populations. At the same time, the gap between student preparedness and the demands of tertiary study has widened. In particular, a lack of mathematical readiness represents a major pedagogical challenge for tertiary educators of quantitative disciplines.

This talk will explore the ways in which diagrams can provide an alternative analytical framework for teaching a quantitative discipline (introductory physics) in a context where prior mathematical experience cannot be automatically assumed (pre-university foundation studies). The extent to which diagrams can be used as an adjunct (or substitute) to traditional approaches (algebra) will be discussed.

Sunday 9am-3pm

Thoughts on Diagrammatic Reasoning by Practitioners of Diagrammatic Reasoning

James Juniper

The paper discusses the pedagogical motivation for the Workshop before considering some philosophical aspects of Diagrammatic Reasoning (DR) in the Digital Economy. The next section of the paper defines DR and provides a brief genealogy before moving on to examine some subsequent developments in the field. One feature of string diagrams—their power of abbreviation—is then discussed. The views of DR practitioners, as expressed "in their own words", are then reviewed across a variety of application domains. The paper concludes after a brief review of some opportunities for the future development of DR.

Inductive reasoning or mathematical induction? Investigating mathematical thinking through 'visual proofs'

Rafael Núñez

Formal deductive proofs are essential for the validation of bodies of knowledge (i.e., theorems) in modern mathematics. The elaboration of proofs involves reasoning with concepts that are often intertwined with complex supporting notations and algorithms associated with them. How to investigate the principles underlying these concepts without getting over-entangled with notational features? Here, using 'visual proofs' we investigate fundamental differences between standard inductive reasoning and reasoning by mathematical induction. While the former is acceptable means of discovery (but not justification), the latter—a formal, deductive proof strategy—can be used to show that a theorem is *necessarily* true for all natural numbers. I will present a study conducted in our lab by Josephine Relaford-Doyle in which we examine the nature of the conclusions drawn from a visual proof by induction. We find that, while most university-educated viewers demonstrate a willingness to generalize the statement to nearby cases not depicted in the image, only viewers who have been trained in formal proof strategies show significantly higher resistance to the suggestion of large-magnitude counterexamples to the theorem. We conclude that for most university-educated adults without proof-training the image serves as the basis of a standard inductive generalization and does not provide the degree of certainty (necessity) required for mathematical proof. With the help of 'visual proofs' as a methodological tool, it is possible to empirically show that, despite its name, mathematical induction is not standard induction, but a deductive proof strategy that requires technical training.

The Versatile Power of Simple Diagrams in Mathematics

Keith Devlin

I'll focus on one particular diagram that has proved to be powerful from both the most abstract realms of pure mathematics to the application of mathematical thinking to messy, day-to-day phenomena.

4 Organisers

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