Simple analytical approximations to farfield or short ship waves

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Highlight

The analytical approximations to farfield and/or short ship waves recently given in [1-4] are summarized.

[1] H. Wu, J. He, Y. Zhu, F. Noblesse. The Kelvin–Havelock–Peters farfield approximation to ship waves, Europ. J Mech./B Fluids 70 (2018) 93-101.

[2] H. Wu, J. He, H. Liang, F. Noblesse. Influence of Froude number and submergence depth on wave patterns, Europ. J Mech./B Fluids, 75 (2019).

[3] H. Wu, J. He, Y. Zhu, C.-J. Yang, W. Li, F. Noblesse. Neumann–Michell theory of short ship waves, Europ. J Mech./B Fluids 72 (2018) 601-615.

[4] H. Wu, J. Wu, J. He, F. Noblesse. Wave profile along a ship hull, short farfield waves, and broad inner Kelvin wake sans divergent waves, submitted.

Kelvin-Havelock-Peters (KHP) approximation to farfield ship waves, and application to illustrate the influence of the Froude number or/and the submergence depth on wave patterns

Farfield ship waves are given by a linear superposition of elementary waves. The integrand of this classical integral representation of ship waves oscillates very rapidly in the farfield, and farfield ship waves therefore cannot be easily evaluated numerically. However, three classical analytical approximations given by Kelvin in 1891, Havelock in 1908 and Peters in 1949 can be used to analyze farfield ship waves. Kelvin's and Peters' farfield approximations are only valid inside or outside the cusp angles $\psi = \pm\psi^K \approx \pm 19^\circ 28'$ of the Kelvin wake and are singular at the cusps. Havelock's approximation is only valid at the cusps. A well-known global analytical approximation, valid inside and outside the cusps, has been given by Chester, Friedman and Ursell (CFU) in 1957. This relatively complicated analytical approximation, somewhat simplified by Borovikov in 1994, involves the Airy functions Ai and Ai$'$, and does not provide a simple explicit decomposition of ship waves into transverse and divergent waves. References to these classical farfield approximations can be found in [1].

A simple analytical approximation that combines the Kelvin, Havelock and Peters classical asymptotic approximations is given in [1]. This approximation, called Kelvin–Havelock–Peters (KHP) approximation in [1,2], is nearly equivalent to the Kelvin and Peters approximations inside or outside the cusps. However, the KHP approximation is finite and agrees with Havelock's approximation at the cusps $|\psi| = \psi^K$ of the Kelvin wake, where Kelvin's and Peters' approximations are singular as was already noted. The KHP approximation given in [1,2] is less accurate than the CFU approximation, but is more realistic than the Kelvin and Peters approximations, in the vicinity of the cusps.

The KHP approximation given in [1-2] is a fully-analytical approximation that only involves elementary functions, like the Kelvin, Havelock and Peters approximations. Indeed, the KHP approximation is as simple as these three basic approximations. For ray angles $|\psi| \leq \psi^K$, the KHP approximation yields a useful explicit decomposition of farfield ship waves into transverse and divergent waves, as in Kelvin's original approximation. Thus, the KHP farfield approximation is well suited for analytical and numerical studies of the transverse and divergent waves in the vicinity of the cusps of the Kelvin wake.

Indeed, the KHP approximation given in [1] is applied in [2] to analyze the wave patterns of a surface-piercing monohull ship, a submerged point source, and a fully-submerged body. Specifically, the KHP approximation is used in [2] to illustrate the influence of the Froude number $F \equiv V/\sqrt{gL}$ based on a ship length L, and—for a body submerged below the free surface—the influence of the Froude number $F_\Delta \equiv V/\sqrt{g\Delta}$ based on the submergence depth Δ, on wave patterns in deep water. The influence of the Froude number F is illustrated for a free-surface piercing monohull ship; and the influence of the Froude number F_Δ is illustrated for a point source submerged at a depth Δ below the free surface. The combined influences of F and of the submergence depth $\Delta/L \equiv F^2/F_\Delta^2$ are illustrated for a fully-submerged body.

These numerical illustrations show that a ship wave pattern—which does not depend on L or Δ within Kelvin's analysis for a ship modeled as a 1-point wavemaker—in fact is greatly influenced by the Froude numbers F and F_Δ. In particular, at high Froude numbers, e.g. at $F = 1.5$, a ship wave pattern mostly contains divergent waves that are most apparent *well inside* the cusps of the Kelvin wake due to interferences between the dominant waves created at a ship bow and stern, as is explained and illustrated in [5-12]. A very different wave pattern is obtained at low Froude numbers, e.g. at $F = 0.2$, for which the dominant waves are found *outside* the cusps of the Kelvin wake. Indeed, [2] shows that wave patterns due to surface-piercing or submerged bodies can differ greatly from Kelvin's classical pattern of transverse and divergent waves found inside a 39° wedge aft of a ship, as is illustrated in Fig.1. Indeed, transverse *and* divergent waves cannot be clearly identified for many of the wave patterns depicted in [2] because one of these two sets of waves is usually dominant.

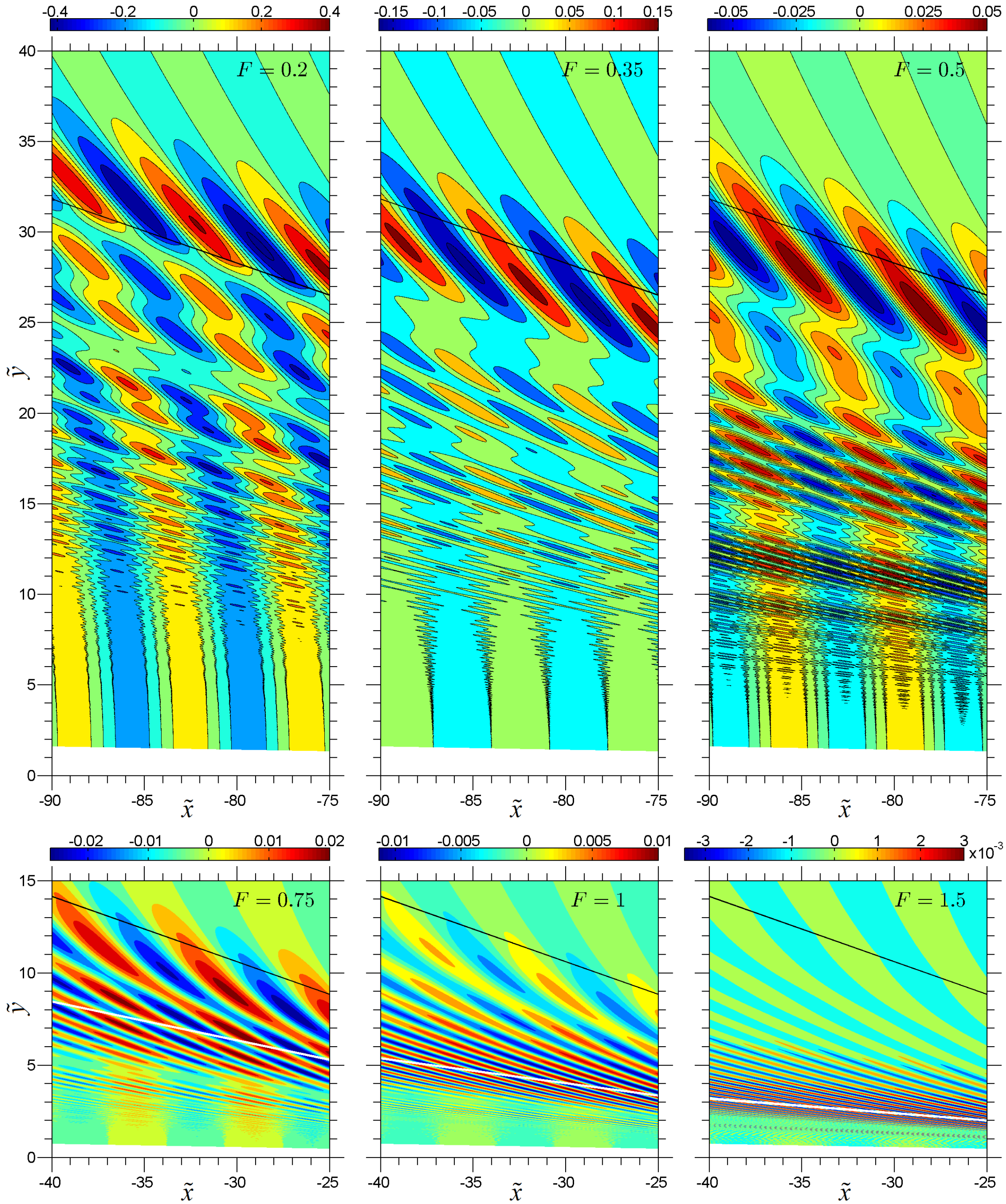

Figure 1: Colored contour plots of the free-surface elevation Eg/V^2 within patches $(\widetilde{X},\widetilde{Y})g/V^2$ located far behind a free-surface piercing monohull ship at six Froude numbers $F = 0.2$, 0.35, 0.5 (top) and $F = 0.75$, 1, 1.5 (bottom). Thin solid black lines mark the cusp $\psi = \psi^K \approx 19^\circ 28'$ of Kelvin's wake. For the high Froude numbers $F = 0.75, 1$ and 1.5, the highest peak of the wave-amplitude function a^D related to divergent waves is marked via a thick solid white line. The dominant waves are located *outside* the cusp line $\psi = \psi^K$ at the low Froude number $F = 0.2$, but mainly consist of short divergent waves that are most apparent *well inside* the cusp line at the high Froude numbers $F = 1$ and (especially) 1.5 due to interferences between the dominant waves created by the bow and the stern of the ship.

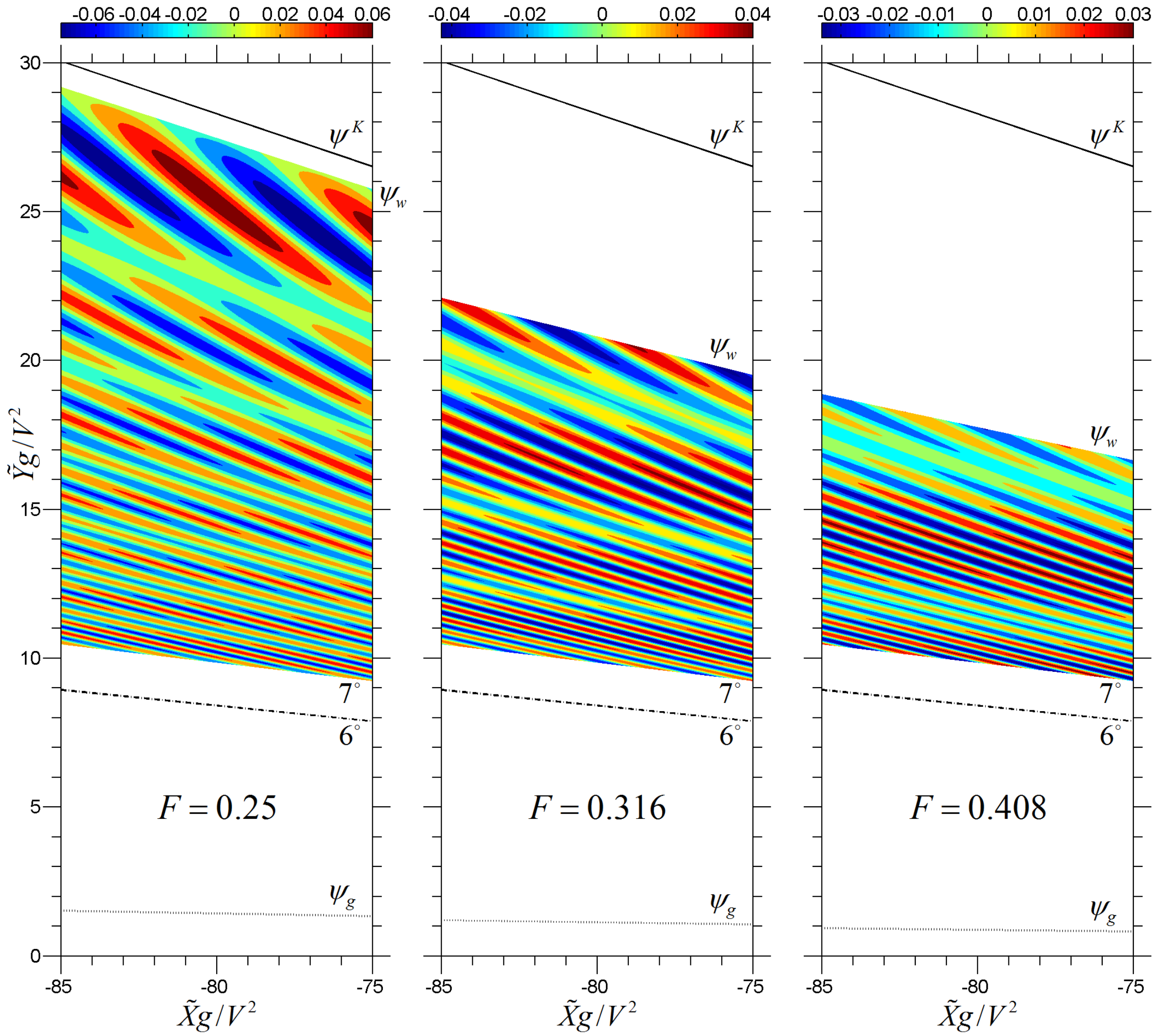

Figure 2: Colored contour plots of the divergent waves within the region $-85 \le \widetilde{X}g/V^2 \le -75$ located far behind the Wigley hull at the Froude numbers $F = 0.25$ (left), 0.316 (center) and 0.408 (right). Solid lines mark the Kelvin cusp line $\psi = \psi^K$, dotted lines define the region $\psi_g \le \psi$ where the waves created by a full-scale ship of length $200\,\mathrm{m} \le L$ are not influenced by surface tension, and dashed-dotted lines mark the ray angle $\psi = 6^\circ$. Divergent waves are not shown for $\psi_w < \psi \le \psi^K$ where the short wave approximation given in [3] and applied in [4] is not valid, and for $\psi < 7^\circ$ where they are too steep to exist. Divergent waves are then only depicted for ray angles $7^\circ \le \psi \le \psi_w$, where they can exist and the short wave approximation given in [3,4] is realistic.

Neumann-Michell theory of short ship waves, wave profile along a ship hull, and broad inner Kelvin wake where divergent waves are too steep to exist

The classical Fourier–Kochin (FK) representation of ship waves expresses ship waves as a linear Fourier superposition of elementary waves. Within the Neumann–Michell (NM) theory considered in [3], the amplitudes of the elementary wave components in this Fourier superposition are given by a wave-amplitude function that is determined in terms of the flow velocity at the mean wetted ship hull surface. In the short wave limit $k \to \infty$, Laplace's method can be applied to approximate the hull-surface integral as a line integral around the mean ship waterline. Moreover, the method of stationary phase can be applied to obtain analytical approximations. The short-wave approximations given in [3] are valid for all—divergent and transverse— short waves created by a ship at a low Froude number, as well as for short divergent ship waves at all—low or high—Froude numbers.

These asymptotic approximations to the wave-amplitude function in the FK representation of ship waves associated with the NM theory provide useful insight into the influence of the Froude number, and the shape of the ship hull surface in the vicinity of the ship waterline, on short farfield ship waves. In particular, these analytical approximations show that short ship waves are predominantly created by the bow and the stern of the ship, and by particular points of the ship waterline. The location of these particular points, which correspond to points of stationary phase, along the ship waterline does not depend on the Froude number. However, the relative importance of the contributions of the points of stationary phase and of the ship bow and stern depends on the Froude number, and also depends on the shape of the waterline, as well as the flow velocity at the bow and the stern and at the points of stationary phase. Thus, the asymptotic approximations given in [3] show that the relationship between the Froude number, the shape of a ship hull surface (notably the shape of the waterline), and short ship waves is fairly complicated.

Indeed, the short divergent waves created by a high-speed ship, and the related apparent narrow wake caused by wave interferences, cannot be fully understood via the simplified analysis based on the 2-point or (for a catamaran) 4-point ship models considered in [5-7], or via the numerical studies based on Hogner's hull-surface distributions of sources and sinks considered in [8-12], for deep water [5,6,8-11] or for uniform finite water depth [7,12]. In particular, the analysis given in [3] explains why the point source and the point sink in the 2-point wavemaker model of a monohull ship should not be located right at the bow and the stern of the ship, but must be separated by a distance that is smaller than the ship length and depends on the Froude number, as in [6,8].

Within the theoretical framework of a nonlinear potential flow theory in which the exact kinematic boundary condition at the ship hull surface and the nonlinear kinematic and dynamic boundary conditions at the free surface are assumed to hold along the contact line between the ship hull surface and the free surface (the ship waterline), these three boundary conditions yield three *algebraic* equations that determine the three flow velocity components u, v, w in terms of the elevation ζ of the free surface along the ship hull. Specifically, the kinematic boundary conditions at the ship hull surface and at the free surface are linear algebraic equations, and the dynamic free-surface boundary condition (Bernoulli's equation) is a quadratic algebraic equation. Thus, these three boundary conditions explicitly determine the three velocity components u, v, w in terms of the wave profile $\zeta(x)$ along the ship hull, as is shown in [13,14].

The expressions for $u(\zeta)$, $v(\zeta)$ and $w(\zeta)$ given in [13,14] can be used in the approximations given in [3] to formally express short ship waves in terms of the shape of the ship hull surface (notably the shape of the waterline) and the wave profile along the ship waterline. Moreover, this relationship between short farfield ship waves and the elevation $\zeta(x)$ of the free surface along the wave profile at the ship hull surface provides insight into the influence of nearfield nonlinearities (arguably most significant at a ship waterline) on farfield ship waves.

This analysis, which provides a direct relationship between the wave profile along a ship hull and the short farfield waves created by the ship via a coupling of the Neumann–Michell linear potential flow theory of short farfield ship waves given in [3] and the nonlinear analysis of the nearfield flow along a ship waterline given in [13,14], is given in [4]. For the Wigley parabolic ship model, considered in [4] for illustration, nearfield nonlinearities are relatively weak and have a limited, although not insignificant, influence on short farfield ship waves. Application to a full-scale Wigley ship also shows that short divergent waves are too steep to exist within a fairly broad region along the ship track. Specifically, [4] shows that divergent waves cannot exist inside a wedge with angle roughly equal to 13°, i.e. about a third of Kelvin's 39° ship wake angle, as is depicted in Fig.2.

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