34^{th} *International Workshop on Water Waves and Floating Bodies*
7^{th}-10^{th} *April 2019, Noah's on the Beach, Newcastle, Australia*

On wave diffraction-radiation by bodies with porous thin plates

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Introduction

Thin plates are commonly known by their capability to induce additional damping for marine and offshore structures (bilge keel, floater skirts,...). In addition, to enhance energy dissipation, one of the practical solutions is to add perforated plates instead of impermeable ones to increase flow separation and reduce wave exciting loads. It can also be used to tune the natural frequency of a number of dynamical devices such as Tuned Liquid Dampers (TLD) [5]. Recently, the interest to this subject has increased and discussed on many occasions during the last previous workshops [3][5][6].

The main purpose of the present work is to provide a general formulation for wave diffraction-radiation by bodies with thin porous plates using Boundary Integral Equation Method (BIEM). The Boundary Value Problem (BVP) has been formulated within linear potential flow theory and generalized modes approach; making it easy to extend to flexible bodies/plates with arbitrary shape. A linear porosity condition has been considered for the plate following [2]. To validate the numerical implementation, a basic configuration of bottom mounted cylinder with porous ring plate has been selected. The (BVP) problem has been solved analytically using an appropriate eigenfunction expansion technique and compared to (BIEM) solution. Excellent agreement was obtained.

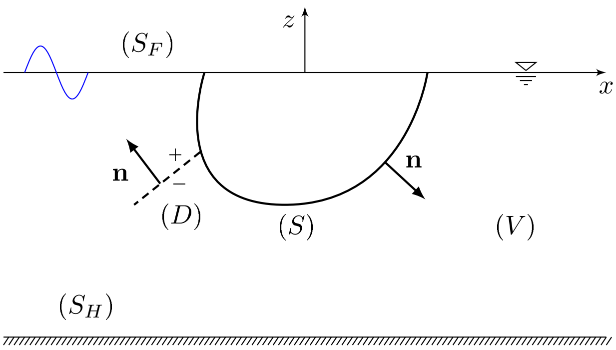

Figure 1: Domain definition

Mathematical model

Linear potential flow theory is used here, the hydrodynamic problem is formulated in frequency domain within generalized modes approach. $(\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z)$ denotes the Cartesian coordinate system and $(\mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_z)$ the cylindrical one with the z-axis pointing upward and $z = 0$ the undisturbed free surface. ω is the wave frequency, g the gravity, ρ the fluid density, h the water-depth, $\nu = \omega^2/g$ the infinite-depth wave number and k_0 the finite-depth one. For the domain definition, (V) stands for the fluid domain, (S_F) the free-surface, (S) the body surface, (S_H) the seabed and (D) the thin plate surface (figure 1).

Under these assumptions, the first order body motion $\mathbf{H}(\mathbf{x}, \omega)$ can be written as follow:

$$\mathbf{H}(\mathbf{x}, \omega) = \sum_{j=1}^{N} \xi_j(\omega)\mathbf{h_j}(\mathbf{x}) \tag{1}$$

Where N the total number of modes (rigid + elastic), $\mathbf{h_j}$ the j^{th} modal displacement vector and $\xi_j(\omega)$ its complex modal amplitude. Similarly, the total potential is decomposed into an incident part ϕ^I, a diffraction part ϕ^D and N radiation potentials ϕ^{R_j}:

$$\phi^{tot}(\mathbf{x}) = \phi^I(\mathbf{x}) + \phi^D(\mathbf{x}) - i\omega \sum_{j=1}^{N} \xi_j(\omega)\phi^{R_j}(\mathbf{x}) \tag{2}$$

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The diffraction-radiation potential ϕ should satisfy the Laplace equation in the fluid domain, the linear free-surface condition, the body boundary condition, the impermeability condition at the seabed and the Sommerfeld condition at the far field:

$$\begin{cases} \nabla^2\phi = 0 & \mathbf{x} \in (V) \\ -\nu\phi + \phi_z = 0 & \mathbf{x} \in (S_F) \\ \phi_{\mathbf{n}} = \mathbf{v}.\mathbf{n} & \mathbf{x} \in (S) \\ \phi_z = 0 & \mathbf{x} \in (S_H) \\ \sqrt{k_0 r}\,(\phi_r - ik_0\phi) = 0 & r \to +\infty \end{cases} \tag{3}$$

$\mathbf{n}$ is the body normal oriented towards the fluid domain and the subscript $_\mathbf{n}$ stands for the for normal derivative. Furthermore, we consider the plate to be infinitely thin with very fine and numerous pores. Therefore, Darcy's law can be applied. The latter implies that the normal relative velocity is continuous and linearly proportional to the pressure drop through the porous plate surface [2]:

$$\phi_{\mathbf{n}}^+ = \phi_{\mathbf{n}}^- = \mathbf{v}.\mathbf{n} + i\kappa\left(\phi^- - \phi^+\right) = \mathbf{v}.\mathbf{n} - i\kappa\mu \qquad \mathbf{x} \in (D) \tag{4}$$

Where μ is the potential drop and κ the porosity coefficient. The superscript $^+$ is used for the positive side of the plate and $^-$ for the negative side with $\mathbf{n}$ the plate normal vector oriented from the negative to the positive side (figure 1). Finally, the term $\mathbf{v}.\mathbf{n}$ in equations (3) and (4) depends on the problem to be solved: $\mathbf{v}.\mathbf{n} = -\phi_{\mathbf{n}}^I$ for the diffraction problem and $\mathbf{v}.\mathbf{n} = \mathbf{h_j}(\mathbf{x}).\mathbf{n}$ for the radiation problem.

To solve this (BVP), Green function is defined for a couple of points $(\mathbf{x}, \mathbf{x'})$ following [1]:

$$\begin{cases} \nabla^2 G(\mathbf{x},\mathbf{x}') = 4\pi\delta(\mathbf{x}-\mathbf{x}') & \mathbf{x} \in (V) \\ -\nu G(\mathbf{x},\mathbf{x}') + G_z(\mathbf{x},\mathbf{x}') = 0 & \mathbf{x} \in (S_F) \\ G_z(\mathbf{x},\mathbf{x}') = 0 & \mathbf{x} \in (S_H) \\ \sqrt{k_0 r}\,\left(G_r(\mathbf{x},\mathbf{x}') - ik_0 G(\mathbf{x},\mathbf{x}')\right) = 0 & r \to +\infty \end{cases} \tag{5}$$

With $\mathbf{x}$ the field point and $\mathbf{x}'$ the singular point. Applying Green identity to ϕ and G and integrating over both sides of the plate, we can write for $\mathbf{x} \in (S)$:

$$2\pi\phi(\mathbf{x}) + \iint\limits_{(S)} \phi(\mathbf{x}')G_{\mathbf{n}_{\mathbf{x}'}}(\mathbf{x},\mathbf{x}')dS_{\mathbf{x}'} + \iint\limits_{(D)} \mu(\mathbf{x}')G_{\mathbf{n}_{\mathbf{x}'}}(\mathbf{x},\mathbf{x}')dS_{\mathbf{x}'} = \iint\limits_{(S)} (\mathbf{v}.\mathbf{n}_{\mathbf{x}'})G(\mathbf{x},\mathbf{x}')dS_{\mathbf{x}'} \tag{6}$$

The subscript $_{\mathbf{n}_{\mathbf{x}'}}$ stands for the normal derivative with the respect to the singular point. For the plate part, the integral equation to satisfy is obtained by taking the Green identity normal derivative when $\mathbf{x} \in (D)$:

$$-4\pi i\kappa\mu(\mathbf{x}) + \iint\limits_{(S)} \phi(\mathbf{x}')G_{\mathbf{n}_{\mathbf{x}'}\mathbf{n}_{\mathbf{x}}}(\mathbf{x},\mathbf{x}')dS + \iint\limits_{(D)} \mu(\mathbf{x}')G_{\mathbf{n}_{\mathbf{x}'}\mathbf{n}_{\mathbf{x}}}(\mathbf{x},\mathbf{x}')dS = -4\pi(\mathbf{v}.\mathbf{n}_{\mathbf{x}}) + \iint\limits_{(S)} (\mathbf{v}.\mathbf{n}_{\mathbf{x}'})G_{\mathbf{n}_{\mathbf{x}}}(\mathbf{x},\mathbf{x}')dS_{\mathbf{x}'} \tag{7}$$

Finally once the potential found, the hydrodynamic forces are calculated by integrating the pressure (respectively pressure difference) over the body (respectively the plate):

$$F_j^{DI} = -i\omega\rho\left(\iint\limits_{(D)} \mu_D(\mathbf{x})\mathbf{h_j}(\mathbf{x}).\mathbf{n_x}dS_{\mathbf{x}} + \iint\limits_{(S)} \left(\phi^I(\mathbf{x}) + \phi^D(\mathbf{x})\right)\mathbf{h_j}(\mathbf{x}).\mathbf{n_x}dS_{\mathbf{x}}\right) \tag{8}$$

$$A_{ij} + \frac{i}{\omega}B_{ij} = -\rho\left(\iint\limits_{(D)} \mu_{R_i}(\mathbf{x})\mathbf{h_j}(\mathbf{x}).\mathbf{n_x}dS_{\mathbf{x}} + \iint\limits_{(S)} \phi^{R_i}(\mathbf{x})\mathbf{h_j}(\mathbf{x}).\mathbf{n_x}dS_{\mathbf{x}}\right) \tag{9}$$

The pressure integration can be checked either by verifying some hydrodynamic identities such as Haskind relation, or making use of Green identity to recompute wave forces by the mean of a control surface located at the far field. A detailed justification of these identities can be found in [4] for the impermeable body case. In our configuration, similar reasoning to [4] yields to identical relations as those found by Zhao [7].

On the other hand, it is important to note that the integral equations (6) and (7) still valid for a quadratic porosity condition provided to adapt correctly κ [3]. In that case, an iterative scheme has to be set up given the dependency of κ on the normal relative velocity [5]. Hence, the diffraction-radiation problem is solved first then the relative velocity at the center of each panel is evaluated after solving the motion equation. This process is repeated until convergence of motions.

Validation case

In order to validate (BIEM) numerical implementation, a basic configuration has been selected which consists of a fixed bottom mounted circular cylinder with ring porous plate located at $z = -d$ (figure 2). The plate is allowed to move vertically so only heave motion has been considered for the radiation problem. a stands for the cylinder radius and b the plate external radius.

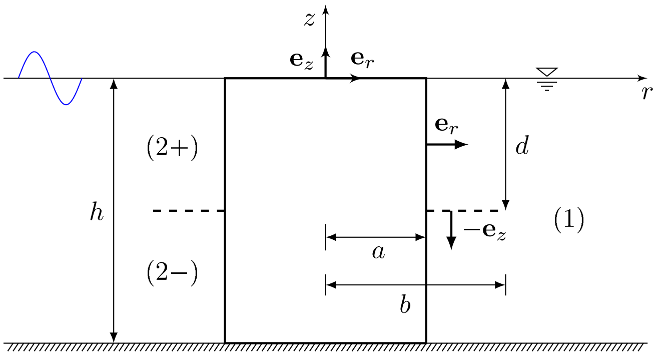

Figure 2: Bottom mounted cylinder with ring porous plate

Eigenfunction expansion method is used here, the fluid domain is divided into 2 region: region 1 ($r \geqslant b$) denoted by the superscript $^{(1)}$ and region 2 ($a \leqslant r \leqslant b$) denoted by the superscript $^{(2)}$. For region 2, the superscript $^{(2+)}$ (respectively $^{(2-)}$) indicates that the potential is valid in the upper part $z \geqslant -d$ (respectively lower part $z \leqslant -d$). The first order incident potential has the form:

$$\phi^I(\mathbf{x}) = \frac{-ig}{\omega} \sum_{m=0}^{+\infty} \epsilon_m i^m J_m(k_0 r) f_0(z) \cos(m\theta) \tag{10}$$

With $\epsilon_0 = 1$ and $\epsilon_m = 2$ for $m > 0$. J_m is the Bessel function of the first kind. The diffraction-radiation potential in the region 1 can be written as:

$$\phi^{(1)}(\mathbf{x}) = \sum_{m=0}^{+\infty} \left[a_{0m} \frac{H_m(k_0 r)}{H_m(k_0 b)} \frac{f_0(z)}{\sqrt{F_0}} + \sum_{n=1}^{+\infty} a_{nm} \frac{K_m(k_n r)}{K_m(k_n b)} \frac{f_n(z)}{\sqrt{F_n}} \right] \cos(m\theta) \tag{11}$$

H_m is the Hankel function of the first kind and K_m the modified Bessel function the second kind. The vertical basis functions $f_n(z)$, which are orthogonal, are defined by:

$$f_0(z) = \frac{\cosh(k_0(z+h))}{\cosh(k_0 h)} \quad , \quad f_n(z) = \frac{\cos(k_n(z+h))}{\cos(k_n h)} \quad , \quad F_n = \int_{-h}^{0} f_n(z)^2 dz \tag{12}$$

Where the wave numbers satisfy $\nu = k_0 \tanh(k_0 h) = -k_n \tan(k_n h)$.

For $a \leqslant r \leqslant b$, the potential $\phi^{(2)}$ is decomposed into two parts $\phi^{(2)} = \phi^{(2P)} + \phi^{(2H)}$ similar to [6]. $\phi^{(2P)}$ is the particular solution which satisfies (3), without the radiation condition, and the porosity condition (4) on the plate. Consequently, $\phi^{(2H)}$ also verifies the same equations but with the following homogeneous boundary conditions:

$$\begin{aligned} \phi_r^{(2H)} &= 0 \qquad r = a \\ \phi_z^{(2H+)} = \phi_z^{(2H-)} &= -i\kappa \left(\phi^{(2H+)} - \phi^{(2H-)} \right) \quad z = -d \end{aligned} \tag{13}$$

Using separation of variables, $\phi^{(2H)}$ can be expressed as:

$$\phi^{(2H)}(\mathbf{x}) = \sum_{m=0}^{+\infty} \left[\sum_{n=1}^{+\infty} b_{nm} \frac{\mathcal{R}_{nm}(r)}{\mathcal{R}_{nm}(b)} \frac{g_n(z)}{\sqrt{G_n}} \right] \cos(m\theta) \tag{14}$$

With:

$$\mathcal{R}_{nm}(r) = H'_m(\lambda_n a) J_m(\lambda_n r) - J'_m(\lambda_n a) H_m(\lambda_n r) \quad , \quad G_n = \int_{-h}^{0} g_n(z)^2 dz \tag{15}$$

g_n is the vertical basis function defined in [6] which takes different form in the upper and the lower domains. Finally, the pressure drop condition in (13) yields to the dispersion relation for the wave numbers λ_n:

$$\lambda_n \sinh(\lambda_n(h-d))\left(\nu\cosh(\lambda_n d) - \lambda_n \sinh(\lambda_n d)\right) = i\kappa\left(\nu\cosh(\lambda_n h) - \lambda_n \sinh(\lambda_n h)\right) \tag{16}$$

This dispersion equation is solved numerically following the method used in [6]. In that way, all the boundary conditions are fulfilled except the velocity and the potential continuity at $r = b$:

$$\phi^{(1)} = \phi^{(2)} \qquad \text{and} \qquad \phi_r^{(1)} = \phi_r^{(2)} \qquad \text{at} \qquad r = b \tag{17}$$

These conditions are used as matching conditions and projected over $f_n(z)$ and $g_n(z)$ to obtain the linear system of the unknown coefficients a_{nm} and b_{nm}:

$$\begin{aligned} \int_{-h}^{0} \phi_r^{(1)} \frac{f_n(z)}{\sqrt{F_n}} dz - \int_{-h}^{0} \phi_r^{(2H)} \frac{f_n(z)}{\sqrt{F_n}} dz &= \int_{-h}^{0} \phi_r^{(2P)} \frac{f_n(z)}{\sqrt{F_n}} dz \\ \int_{-h}^{0} \phi^{(2H)} \frac{g_n(z)}{\sqrt{G_n}} dz - \int_{-h}^{0} \phi^{(1)} \frac{g_n(z)}{\sqrt{G_n}} dz &= -\int_{-h}^{0} \phi^{(2P)} \frac{g_n(z)}{\sqrt{G_n}} dz \end{aligned} \tag{18}$$

Where the particular solution $\phi^{(2P)}$ is the same as the one found for the (BVP) studied in [6]: $\phi^{(2P)} = -\phi^I$ for diffraction problem and $\phi^{(2P)} = \dfrac{i}{\kappa}\mathcal{H}(-z-d)$ for heave problem, $\mathcal{H}$ being the Heaviside function.

Preliminary results and discussions

For numerical tests, we consider a cylinder with $b = 2a$, $d = a$ and $h = 4a$. Also, we introduce the non-dimensional porosity parameter [2] defined by $\kappa_0 = 2\pi\kappa/k_0$. Figures 3 and 4 show vertical exciting force and heave added mass for different κ_0 values. Almost the same results are obtained by (BIEM) and semi-analytical method. Furthermore, wave forces are decreasing with κ_0 as expected. More detailed results will be presented and discussed at the workshop.

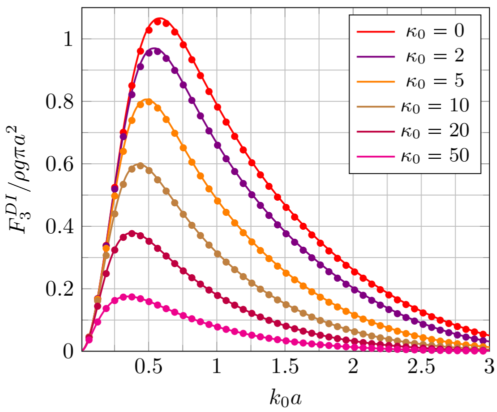

Figure 3: Vertical exciting force, (BIEM) in solid line vs. semi-analytical solution in markers

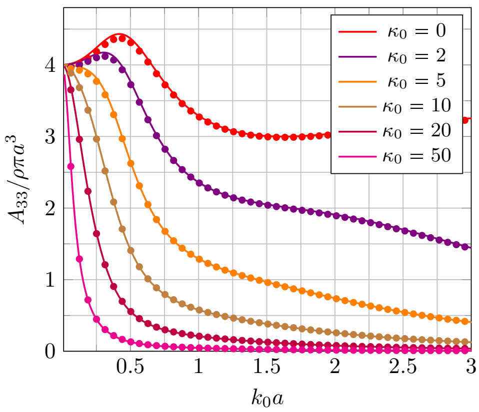

Figure 4: Heave added mass, (BIEM) in solid line vs. semi-analytical solution in markers

References

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