On the periodic waves interacting with linear shear currents

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HIGHLIGHTS:

- The time domain simulation of the periodic waves over linear shear currents is conducted by using strongly nonlinear HLGN wave model.
- Strongly nonlinear waves interacting with strongly linear shear currents are studied.
- The velocity field in following currents and opposing currents are studied.
- The method here is easily extend to solve interaction problem between waves and nonlinear shear currents.

1 INTRODUCTION

Water waves and currents often coexist in the ocean environment. In previous studies, different kinds of wave-current interaction were studied. In the case of a uniform current, vorticity is zero and potential theory could give good results on this topic.

In the case of waves over linear shear current, Tsao (1959) make a classical Stokes expansion, obtained the second-order expressions for wave profile and velocity distribution in a linear shear current. Kishida and Sobey (1988) gave the Stokes third-order solution of periodic waves on a linear shear current problem. Kemp & Simons (1982, 1983), and Swan (1990) conducted experimental study of periodic waves on a linear shear current, and presented some experimental data on wave profile and velocity distribution. Chen et al. (2014) also simulated periodic waves on a linear shear current by their COBRAS model. Son and Lynett (2014) studied the interaction of periodic water waves with linear shear currents through a set of depth-integrated equations. More recently, Touboul et al. (2016) simulated the propagation of periodic waves over a vertically sheared current by deriving an extension of the Mild-Slope equation.

In this abstract, we will use a strongly nonlinear, strongly dispersive model, the High-Level Green-Naghdi model (HLGN model), to investigate the waves over linear shear currents. Strongly nonlinear waves interacting with strongly linear shear currents are studied here. The velocity field in following currents and opposing currents are studied here. The method here is easily extend to solve interaction problem between waves and nonlinear shear currents.

The HLGN model are described in Section 2, boundary condition is introduced briefly in Section 3, test cases are presented in Section 4, and the conclusions we reach are in Section 5.

2 HLGN wave model

The fluid is immiscible and inviscid. Here we consider a two-dimensional case. The mass density is ρ. The free surface and bottom are expressed by $z = \beta(x, t)$ and $z = -h$, respectively.

The mass conservation equation is

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

The momentum conservation equations are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{2}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\left(\frac{\partial p}{\partial z} + \rho g\right) \tag{3}$$

The free surface and bottom boundary conditions are

$$w - \frac{\partial \beta}{\partial t} - u\frac{\partial \beta}{\partial x} = 0 \quad (z = \beta) \tag{4}$$

$$\hat{p} = 0 \quad (z = \beta) \tag{5}$$

$$w = 0 \quad (z = -h) \tag{6}$$

In the high-level Green-Naghdi model (HLGN model), the horizontal velocity and vertical velocity are expressed as a polynomial form as

$$u(x,z,t)=\sum_{n=0}^{K-1}u_n(x,t)z^n, \qquad w(x,z,t)=\sum_{n=0}^{K}w_n(x,t)z^n, \tag{7}$$

where *K* is the level of the HLGN model.

The governing equations of the HLGN model are

$$\frac{\partial\beta}{\partial t}=\sum_{n=0}^{K}\beta^n\left(w_n-\frac{\partial\beta}{\partial x}u_n\right) \tag{8}$$

$$\frac{\partial}{\partial x}(G_n+gS1_n)+nE_{n-1}-\alpha^n\frac{\partial}{\partial x}(G_0+gS1_0)=0 \ \ (n=1,2,\cdots,K) \tag{9}$$

For the algorithms and more details on the HLGN equations, readers are referred to Webster et al. (2011).

3 Boundary condition for wave-current interaction

In this work, we study periodic waves propagating over linear shear currents, see Fig 1.

For the periodic regular waves only (without the current), we can easily obtain the wave elevation $\eta'(x;\, t)$ and velocity coefficients u_n' (n = 0, 1, 2, · · · , K -1) (the primes over quantities represent the HLGN solution for periodic waves in the absence of any current).

The linear shear (background) current u_c can be expressed as $u_c=U_c+U_{c1}z$, the magnitude of current at z = 0 is U_c. See Fig. 1. With different U_c and U_{c1}, we could have the following-current case and the opposing-current case.

In this work, we simulate periodic waves on linear shear currents. The wave-maker boundary uses the linear superposition method, superposing the linear shear current below the HLGN solution of waves only. Fig. 2 presents the boundary conditions used in this work, where we observe that the computational region is divided into three parts.

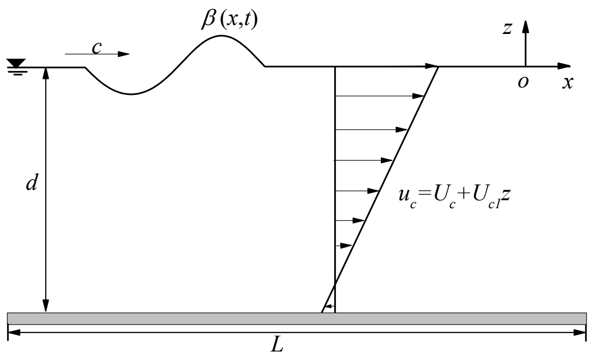


Fig. 1 Periodic regular waves interacting with linear shear currents

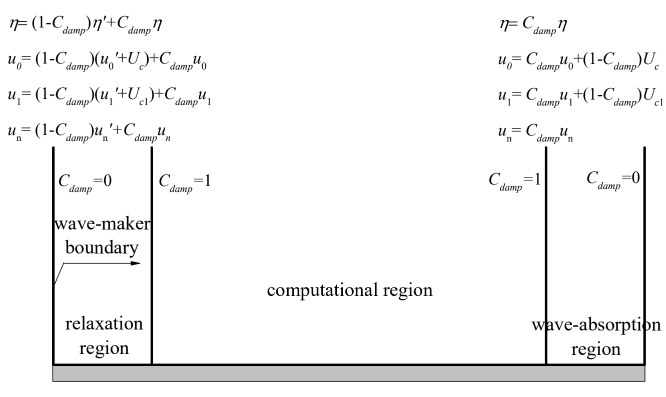


Fig.2 Boundary condition for wave and currents interaction

4 TEST CASES

Some results of periodic waves propagating over linear shear currents are presented here. Both the wave profile and velocity distribution are studied. The relevant parameters are given in Table 1. In this work, H is wave height, T is wave period, d is water depth.

Table 1 Test cases parameters

Case	d(m)	H/d	T(s)	u_c(z) (m/s)	Comparison with
1	0.6	0.1	1.159	0	Chen et al. (2014)
2	0.6	0.09	1.159	$0.24+0.4z$	Chen et al. (2014)
3	0.6	0.078	1.159	$0.54+0.9z$	Chen et al. (2014)
4	0.35	0.18	1.418	$0.42+1.7z$	Swan(1990) Kishida & Sobey(1988) Son & Lynett (2014) Touboul et al. (2016)
5	0.35	0.35	1.42	$-0.5-1.67z$	Swan(1990) Kishida & Sobey(1988) Son & Lynett (2014) Touboul et al. (2016)

After careful self-convergence test, we compare the HLGN results with others' results. For Case 1 shown in Table 1, the HLGN results show very good results on wave elevations with others, see Fig. 3. When waves interacting with currents, there will be interaction between them, and the total velocity field is not a simple sum of wave only velocity field and current only velocity field. For Case 4 and Case 5 in Table 1, the waves are strongly nonlinear waves and the current is also strong. The HLGN results shows good agreement with others, see Fig. 4.

As introduced in the beginning, waves interacting with uniform currents and linear shear currents has been done by many researchers, because the vorticity is zero or constant, so it is easier to solve by using potential theory. But for the case which waves interacting with nonlinear shear currents, the vorticity will vary with time and space.

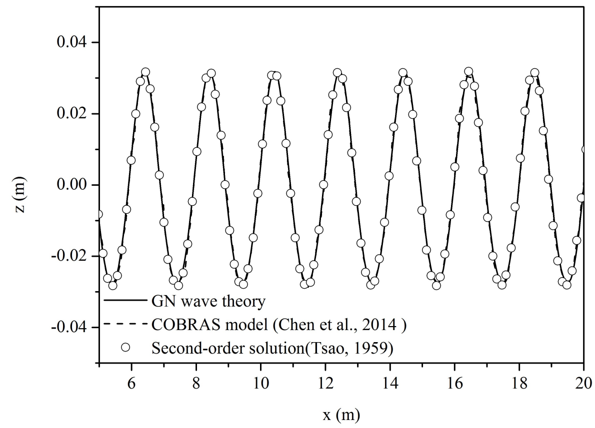
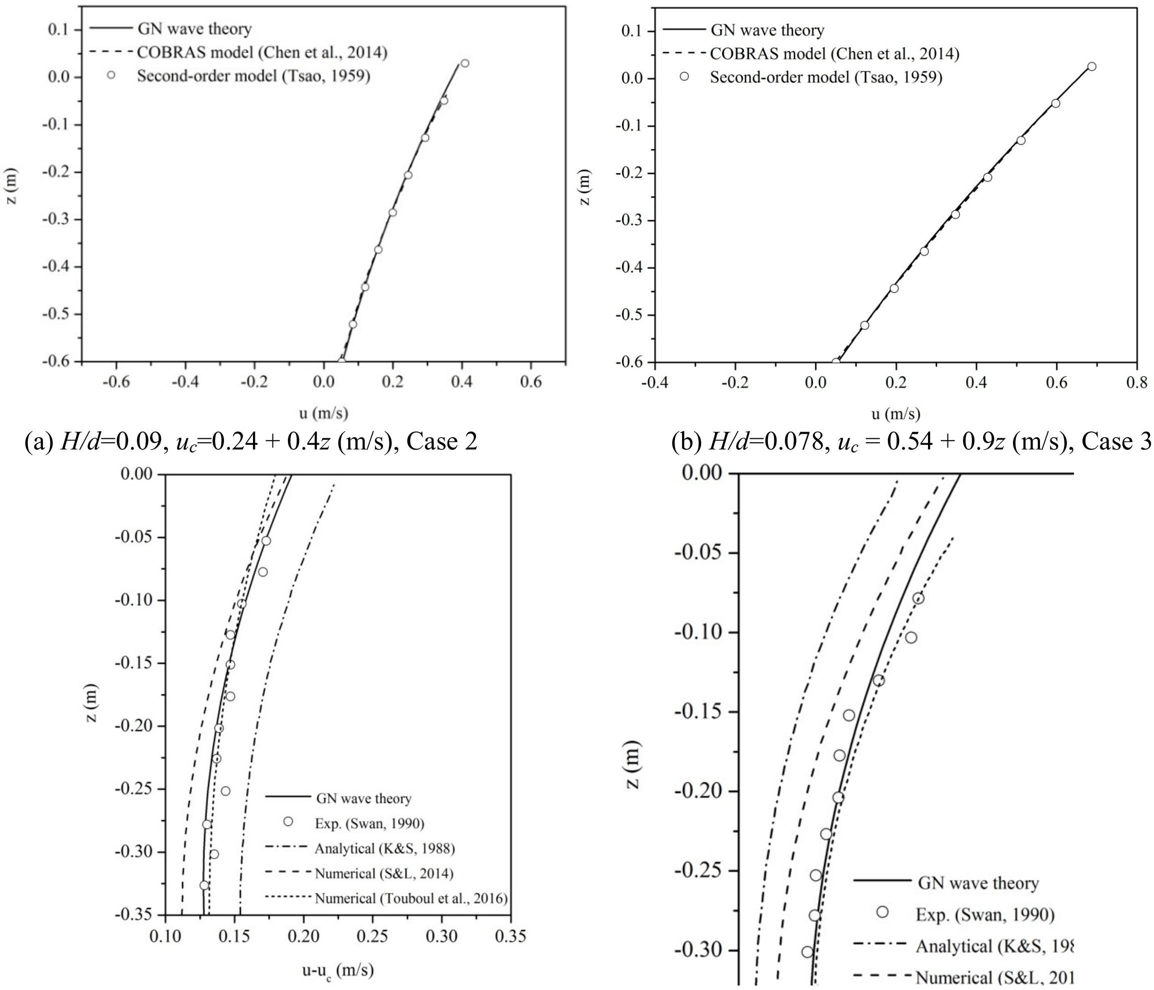


Fig.3 H/d=0.1， u_c=0， Case 1

HLGN models and the method proposed in this work is easy to solve the problem on waves interacting with nonlinear shear currents or arbitrary shear currents. **More results of waves interacting with nonlinear shear currents will be presented at the workshop.**



(a) H/d=0.09, u_c=0.24 + 0.4z (m/s), Case 2 (b) H/d=0.078, u_c = 0.54 + 0.9z (m/s), Case 3

(c) H/d=0.18, u_c=0.42+1.7z(m/s), Case 4 (d) H/d=0.35, u_c=−0.5−1.67z(m/s), Case5

Fig.4 horizontal velocity distribution along water depth, Case 2,3,4 and 5

5 CONCLUSIONS

The time domain simulation of the periodic waves over linear shear currents is conducted by using strongly nonlinear HLGN wave model. Strongly nonlinear waves interacting with strongly linear shear currents are studied. The velocity field in following currents and opposing currents are studied. In all the test cases, the results of HLGN wave models agree well with others' results. HLGN wave models and the method proposed in this work is easy to solve the problem on waves interacting with nonlinear shear currents or arbitrary shear currents.

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