Application of the High-Level Green-Naghdi model on internal solitary waves with a free surface

Z. Wang, B.B. Zhao, W.Y. Duan, T.Y. Zhang

College of Shipbuilding Engineering, Harbin Engineering University, Harbin, China
Email: zhan.wang@hrbeu.edu.cn

HIGHLIGHTS:

- Internal solitary waves with a free surface in a two-fluid system are investigated by the High-Level Green-Naghdi model which considers the free-surface effect (HLGN-FS model).
- Experiments on internal solitary waves in a two-fluid system with a free surface are conducted (ρ_2/ρ_1=0.869, h_2/h_1=1/15).
- The HLGN-FS results on wave profile show good agreements with present experimental data.

1 INTRODUCTION

Among the studies on internal solitary waves, a two-fluid immiscible system is always considered. For some experimental work, such as Segur and Hammack (1982), Kao et al. (1985) and Grue et al. (1999), fresh water and brine are selected. On the other hand, oil and fresh water are also treated as an ideal system for internal solitary waves, such as selected by Koop and Butler (1981), Michallet and Barthelemy (1998) and Kodaira et al. (2016).

Michallet and Barthelemy (1998) and Kodaira et al. (2016) pointed out that if the mass density ratio of the two fluids was close to 1 (e.g., brine and fresh water), the disturbance at the free surface was very small. Thus, free surface can be treated as a rigid lid. Based on this phenomenon, some numerical or analytical methods under the rigid-lid assumption are proposed to study internal solitary waves, such as the methods by solving Euler's equations by Grue et al. (1999), the strongly nonlinear wave model, MCC model derived by Miyata (1985) and Choi and Camassa (1999) and High-Level Green Naghdi model (HLGN-RL model) by Zhao et al. (2016).

While, when the mass density ratio of the two fluids is not close to 1 (e.g., oil and fresh water), the free-surface effect cannot be neglected. Thus, the numerical methods that consider the free surface are needed. Recently, Kodaira et al. (2016) made a comparative study between the MCC model under the rigid-lid assumption (MCC-RL model) and the MCC model that considered the free-surface effect (MCC-FS model). They found that the MCC-FS results matched the experimental data very well on wave profile. The wave profile obtained by the MCC-RL model was wider than that obtained by the MCC-FS model. However, in the MCC-FS model, the horizontal velocity along the fluid column is as constant for each layer (the same as GN-1 model in each layer). Thus, it is a strongly nonlinear, weakly dispersive model. Also, in their experiments, the depth ratio of the upper fluid and lower fluid is only 1/5. For some strongly dispersive cases, the accuracy of the MCC-FS model is unknown.

In this paper, we will use a strongly nonlinear, strongly dispersive model, the High-Level Green-Naghdi model which considers the free-surface effect (HLGN-FS model), to investigate the internal solitary waves in a two-fluid system with a free surface. It is the future work of Zhao and Duan (2013). Also, experiments on internal solitary waves with a free surface are conducted. The depth ratio of the upper fluid and lower fluid is 1/15 and the density ratio is 0.869. As our knowledge, a comparative study between the experimental data and numerical results on internal solitary waves with a free surface in such strongly dispersive case has not been made before.

The two-layer HLGN-FS model are described in Section 2, experiment is introduced briefly in Section 3, test cases are presented in Section 4, and the conclusions we reach are in Section 5.

2 TWO-LAYER HLGN-FS MODEL

Sketch of this physical problem is shown in Fig. 1. Two kinds of fluids are immiscible and inviscid. The mass densities are ρ_2 and ρ_1 for the upper fluid and lower fluid, respectively. h_2 and h_1 are the depths of the upper fluid and lower fluid before the free surface and interface are disturbed. The free surface, interface and bottom are expressed by $z = \gamma(x, t)$, $z = \beta(x, t)$ and $z = -h_1$, respectively.

In the one-layer High-Level Green-Naghdi model (HLGN model), the horizontal velocity and vertical velocity are expressed as a polynomial form as

$$u(x,z,t) = \sum_{n=0}^{K-1} u_n(x,t) z^n, \qquad w(x,z,t) = \sum_{n=0}^{K} w_n(x,t) z^n, \tag{1}$$

where K is the level of the GN model.

Following Zhao and Duan (2013), the governing equations of the HLGN-FS model are

$$\frac{\partial \beta}{\partial t}=\sum_{n=0}^{K^L}\beta^n\left(w_n^L-\frac{\partial \beta}{\partial x}u_n^L\right), \tag{2}$$

$$\frac{\partial \gamma}{\partial t}=\sum_{n=0}^{K^U}\gamma^n\left(w_n^U-\frac{\partial \gamma}{\partial x}u_n^U\right), \tag{3}$$

$$\frac{\partial}{\partial x}\left(G_n^L+gS1_n^L\right)+nE_{n-1}^L-\left(-h_1\right)^n\frac{\partial}{\partial x}\left(G_0^L+gS1_0^L\right)+\left(\beta^n-\left(-h_1\right)^n\right)\frac{\rho_2}{\rho_1}\frac{\partial}{\partial x}\left(G_0^U+gS1_0^U\right)=0, \tag{4}$$

$$\frac{\partial}{\partial x}\left(G_n^U+gS1_n^U\right)+nE_{n-1}^U-\beta^n\frac{\partial}{\partial x}\left(G_0^U+gS1_0^U\right)=0, \tag{5}$$

where n=1,2,..., K^L in Eq. (4) and n=1,2,..., K^U in Eq. (5). g is gravity acceleration, and the superscript L and U refer to the variables in the lower layer and upper layer, respectively. GN-K^L-K^U denotes which levels are used in the two layer fluid simulations. For the algorithms and more details on the HLGN equations, readers are referred to Webster et al. (2011).

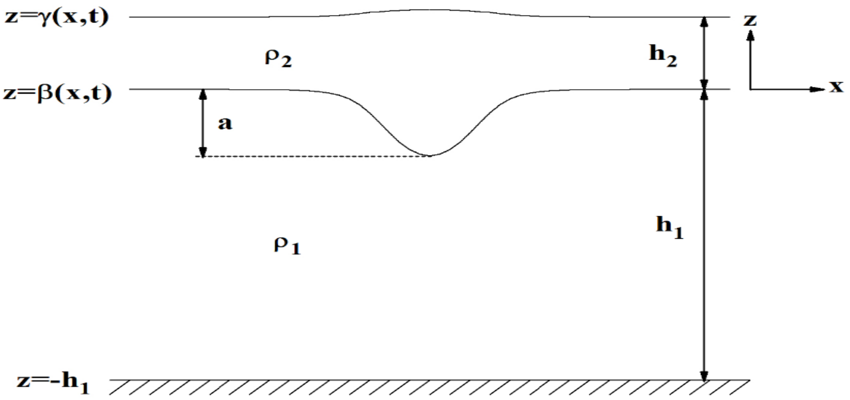


Fig.1 Sketch of the internal solitary waves with a free surface.

3 EXPERIMENT

We conducted some experiments at Harbin Engineering University. Sketch of the experiments is shown in Fig. 2. The mass density of the upper fluid (silicone oil) is 866.5kg/m^3 and the mass density of the lower fluid (fresh water) is 997.5kg/m^3. The depth ratio of the upper fluid and lower fluid is 1/15.We select gravity collapse method to generate internal solitary waves. A grid plate is used to help measure the wave profile. We can obtain the internal solitary waves of different amplitudes by changing the depth ratio of the two kinds of fluids in the collapse zone. In the following section, we will show the experimental results on wave profile for the internal solitary waves of a/h_2=-1.615 and -2.364. For each case, experiments are conducted twice.

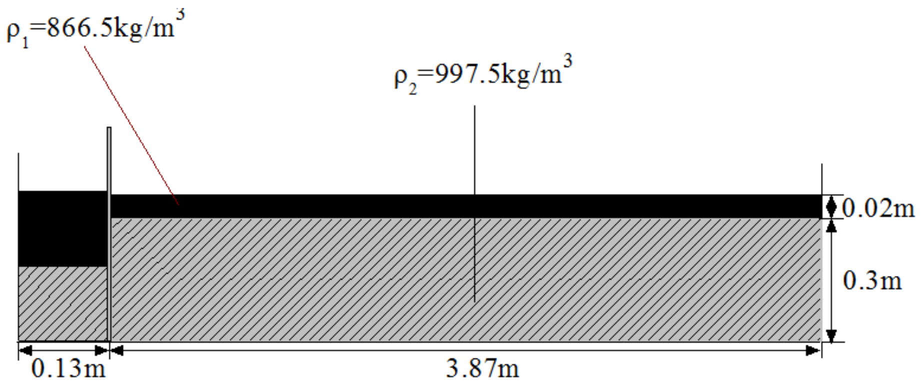
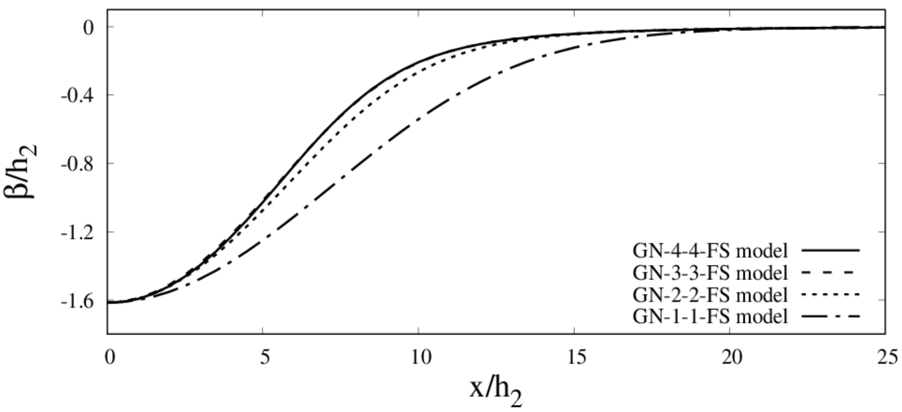


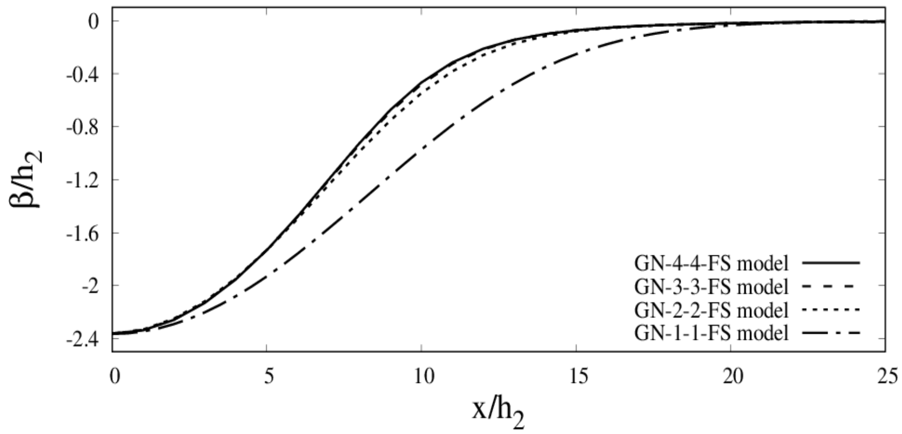
Fig.2 Sketch of the experiments.

4 TEST CASES

In this section, we will use the HLGN-FS model to calculate the internal solitary waves of a/h_2=-1.615 and -2.364. It is necessary to determine which level of the GN-FS models can be used to obtain the converged results firstly. Fig. 3(a) and Fig. 3(b) show the wave profiles obtained by different level GN-FS models for the cases of a/h_2=-1.615 and -2.364, respectively. We find that the GN-2-2 results and GN-3-3 results show some differences. Instead, the GN-3-3 results and the GN-4-4 results show very good agreement. Thus, we can use the GN-3-3 model to obtain the converged results on wave profile.



(a) a/h_2=-1.615

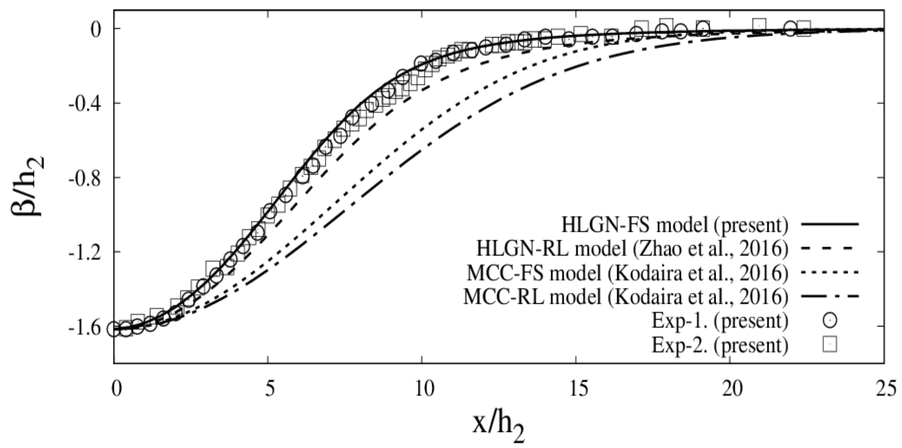


(b) a/h_2=-2.364

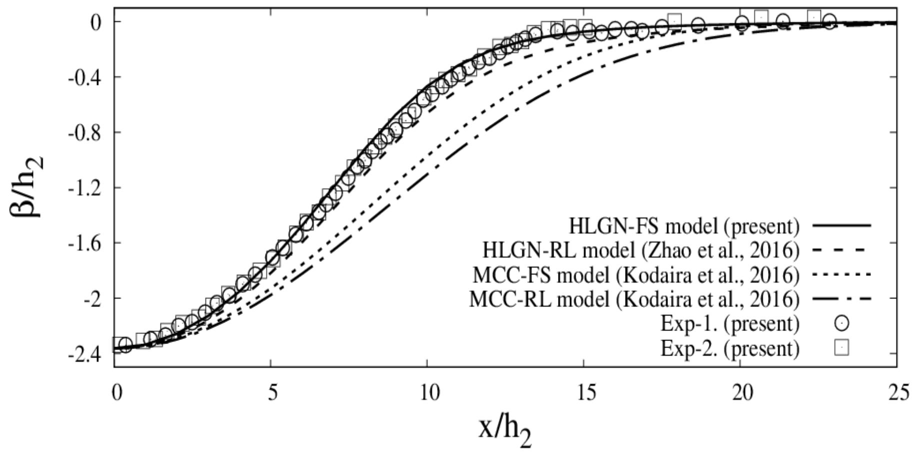
Fig.3 Self-convergence test for the HLGN-FS model

Next, we compare our converged HLGN-FS results with the experimental data, as shown in Fig. 4. Also, the converged HLGN-RL results, MCC-FS results and MCC-RL results are shown for comparison. We find that the converged HLGN-FS results on wave profile show better agreements with experimental data than other models' results. It is found that if the density ratio of the two fluids is not close to 1 (e.g., ρ_2/ρ_1=0.869 in present experiments), the models under the rigid-lid assumption cannot be used to obtain the internal solitary waves with a free surface accurately. Meanwhile, for the strongly dispersive cases (e.g., h_2/h_1=1/15 in present experiments), the weakly dispersive wave models, such as MCC-FS model, also cannot describe the internal solitary waves accurately.

The velocity fields, wave speed and other test cases for the internal solitary waves with a free surface in a two-fluid system will be presented at the workshop.



(a) a/h_2=-1.615



(b) a/h_2=-2.364

Fig.4 Comparisons between the numerical results and experimental data

5 CONCLUSIONS

Internal solitary waves in a two-fluid system with a free surface are investigated both experimentally and numerically. In the experiments, the density ratio ρ_2/ρ_1=0.869 and depth ratio is h_2/h_1=1/15. We generate the large-amplitude internal solitary waves of a/h_2=-1.615 and -2.364.

The HLGN-FS model (the free-surface effect is considered) is used to simulate internal soliatry waves. Results show that the converged HLGN-FS results show better agreement with the experimental data on wave profile than other models' results due to fact that the HLGN-FS model is a strongly nonlinear, strongly dispersive model.

ACKNOWLEDGMENTS

The second and third authors' (B.B. Zhao and W.Y. Duan) work is supported by the National Natural Science Foundation of China (Nos. 11572093, 51490671), International Science and Technology of Cooperation Project sponsored by Nation Ministry of Science and Technology of China (No. 2012DFA70420), the special Fund for Basic Scientific Research of Central Colleges (Harbin Engineering University) and High-Tech Ship Research Projects Sponsored by the Ministry of Industry and Information Technology (MIIT) of China.

REFERENCES

Segur, H., & Hammack, J. L. (1982). Soliton models of long internal waves. J. Fluid Mech., 118, 285-304.

Kao, T. W., Pan, F.-S., & Renouard, D. (1985). Internal solitons on the pycnocline: generation, propagation, and shoaling and breaking over a slope. J. Fluid Mech., 159, 19-53.

Grue, J., Jensen, A., Rusås, P. -O., & Sveen, J. K. (1999). Properties of large-amplitude internal waves. J. Fluid Mech., 380(2), 257-278.

Koop, C. G., & Butler, G. (1981). An investigation of internal solitary waves in a two-fluid system. J. Fluid Mech., 112, 225-251.

Michallet, H. & Barthelemy, E. (1998). Experimental study of interfacial solitary waves. J. Fluid Mech., 366, 159-177.

Kodaira, T., Waseda, T., Miyata, M. & Choi, W. (2016). Internal solitary waves in a two-fluid system with a free surface. J. Fluid Mech., 804, 201-223.

Miyata, M. (1985). An internal solitary wave of large amplitude. La Mer, 23, 43-48.

Choi, W., & Camassa, R. (1999). Fully nonlinear internal waves in a two-fluid system. J. Fluid Mech., 396(10), 1-36.

Zhao, B. B., Ertekin, R. C., Duan, W. Y., & Webster, W. C. (2016). New internal-wave model in a two-layer Fluid. J. Waterway, Port, Coastal, Ocean Eng., 04015022.

Zhao B.B., & Duan W.Y. (2013). GN equations to describe internal solitary waves in two-layer fluid. Proc. 28th IWWWFB, Marseille (France).

Webster, W. C., Duan, W. Y., & Zhao, B. B. (2011) Green-Naghdi theory, Part A: Green-Naghdi (GN) equations for shallow water waves. J. Marine Science and Application, 10(3), 253-258.