Initial experimental validation of a pressure impulse model for a vertical circular cylinder

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1 Introduction

The slamming force calculation for wave impacts on offshore structures is an important step in the design process. Several engineering models are developed and widely used in the industry. Such models usually focus on the peak pressure of the impact and the time series (c.f. Wagner, 1932, Goda, Haranaka, and Kitahata (1966), Cointe and Armand (1987), Wienke and Oumeraci (2005) and Hansen and Kofoed-Hansen (2017)). However, since the duration of the impact is usually very short, the time integrated force impulse may be sufficient for an accurate response prediction. This motivated us to develop a pressure impulse model for impact of a wave on a vertical surface piercing cylinder based on the existing model of Cooker and Peregrine (1995). Relevant work in this area includes the 3D model of Chatjigeorgiou, Korobkin, and Cooker (2017). Our pressure-impulse model is presented in Ghadirian and Bredmose (2019), developed in successive steps from the 2D wall solution of Cooker and Peregrine over a 3D box impact, an axisymmetric cylindrical impact and finally the finite-angle cylindrical impact. While the previous paper compared the model results to the impulse of a CFD-generated impact, the present paper contains our initial validations against experimental data.

2 The pressure-impulse model for vertical circular cylinders

The fluid domain and its boundary conditions are shown in figure 1 (a). The domain is wedge-shaped in the azimuth direction with limits $-\theta_{max} \leq \theta \leq \theta_{max}$ and is divided into heights above and below $z = -\mu H$. At the time of impact, the upper part is approaching the cylinder with velocity $U\cos(\theta)$ in the negative radial direction. At the boundaries $\theta = \pm\theta_{max}$, $z = 0$ and $r = b$ the condition $P = 0$ is satisfied. The Laplace equation is solved in the cylindrical coordinate system to yield

$$P \quad = \quad \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(A_{mn}\cos(L_m\theta/\theta_{max})\sin(k_n\frac{z}{H})\frac{I_{q_m}(k_n\frac{r}{H})+\alpha_{mn}K_{q_m}(k_n\frac{r}{H})}{\partial_r\left(I_{q_m}(k_n\frac{r}{H})+\alpha_{mn}K_{q_m}(k_n\frac{r}{H})\right)_{r=a}}\right), \tag{1}$$

where $q_m = L_m/\theta_{max}$ is the order of the Bessel functions, $k_n = (n-1/2)\pi$ and $L_m = (m-1/2)\pi$. Further α_{mn} is chosen such that $P = 0$ at $r = b$ and

$$A_{mn} = \frac{2\rho U}{\theta_{max}}\frac{1-\cos(k_n\mu)}{k_n}\int_{-\theta_{max}}^{\theta_{max}}\cos(\theta)\cos(L_m\theta/\theta_{max})d\theta dz. \tag{2}$$

The non-dimensional pressure impulse depends on the normalized outer radius, b/H, impact height μ, cylinder radius a/b and the maximum impact angle θ_{max}:

$$\frac{P}{\rho U H}\left(\frac{r}{H},\theta,\frac{z}{H}\right) = f\left(\frac{b}{H},\mu,\frac{a}{b},\theta_{max}\right). \tag{3}$$

The effect of each parameter was investigated by Ghadirian and Bredmose (2019). It was concluded that θ_{max} was the only parameter not determinable from the incident wave. A value of $\pi/4$ was found to provide a good match to the CFD impact analysed. In figure 1 (b), an example distribution of the pressure impulse on the cylinder is shown for $b/H = 5$, $\mu = 0.5$, $a/b = 0.5$ and $\theta_{max} = \pi/4$.

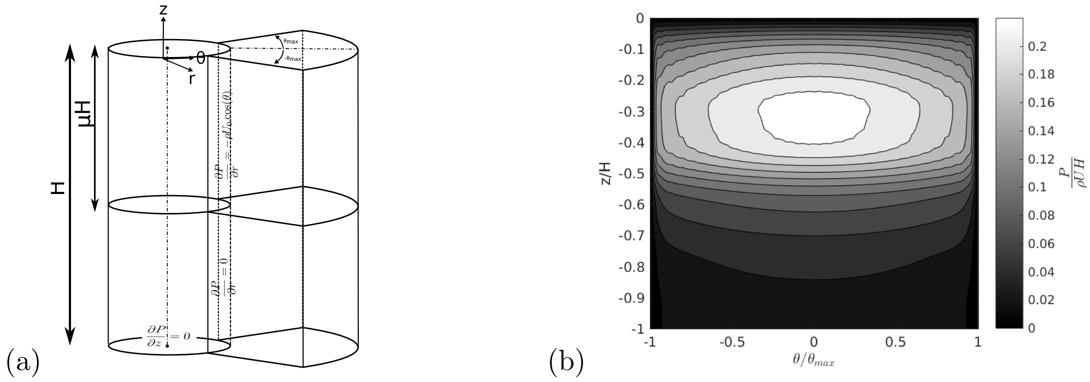


Figure 1: (a) Definition sketch for wedge-shaped 3D impact on a vertical cylinder. (b) Example of the pressure impulse distribution on the cylinder wall.

3 Slamming detection and impulse validation

The experiments were performed at DHI Denmark as part of the DeRisk project (Bredmose et al., 2016) in scale 1:50. The monopile diameter was 7 m and the water depth in the presented results was 20 m. The presented results include tests with peak period of 15 s while the significant wave height was either 6.8 m or 7.5 m, all in full scale. Two sets of tests with and without directional spreading are used here.

Slamming events were detected by peak separation and inspection for abrupt increases in the measured inline force histories. The inline force history was next Butterworth filtered to remove the structural response from the time series. Hereafter the “force time series” term refers to the time series excluding the structural response. We report here our initial model comparison for 6 events chosen this way, denoted case 1-6 in the following.

Once the events for analysis were selected, the slamming impulse was determined by time integration of the slamming induced force-peak that occurs on top of the non-slamming force history. This requires a robust determination of the non-slamming force history. Three methods were applied, based on i) the instant of an abrupt slope change in the force time series; ii) calculation of the MacCamy-Fuchs corrected Morison force from the free surface elevation measured close to the cylinder and iii) further filtering of the the force time series. For the present events, i) was found to give an over-prediction of the measured impulse and ii) was found to give under-predicted impulse values when compared to the associated pressure impulse model results. The best results were obtained with the filtering approach (iii). Figure 2 shows the force time series, the filtered force time series without the slamming and the free surface elevation at 0.2 m up stream from the cylinder for case 6.

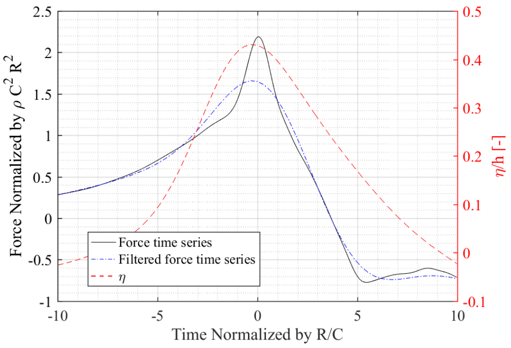
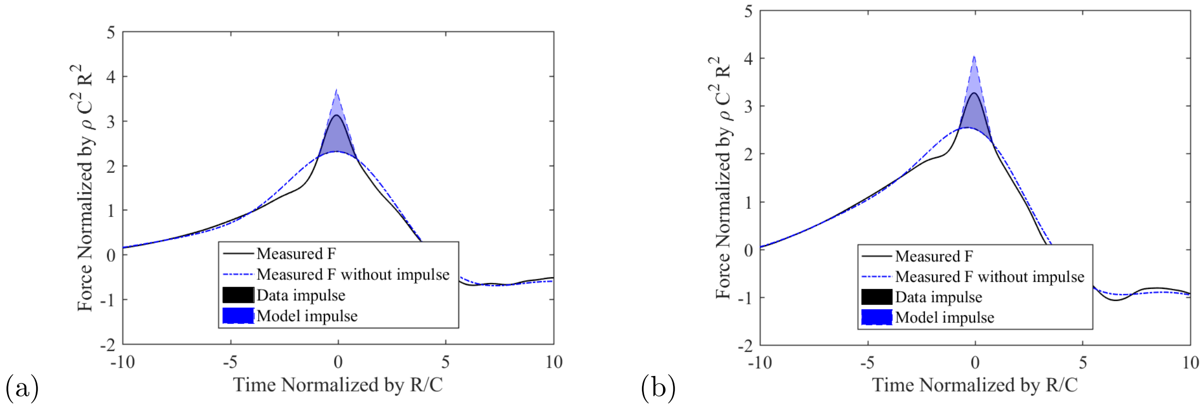


Figure 2: Sample time series of the inline force, filtered inline force without the slamming effect and free surface elevation at 0.2 m upstream from the cylinder center for case 6.

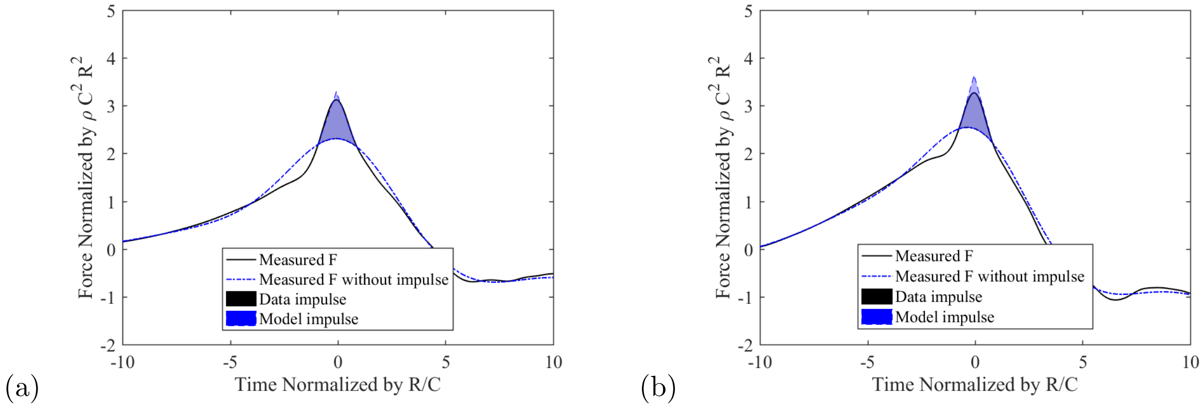
As can be seen from (3), parameters should be detected for each case to calculate the impulsive force by the model. The total depth of the water from the free surface, H, is calculated using the free surface measurement at 0.2 m upstream of the cylinder center. The length of the slamming water,

b, was detected from several free surface measurements at the instant of the maximum inline force. As concluded by Cooker and Peregrine (1995) and Ghadirian and Bredmose (2019) the parameter b has less importance when it is larger than 10 times the radius of the cylinder which is the case here. The inner diameter, a, is equal to the radius of the cylinder. The ratio of the slamming height of the water to the total water height, μ, was determined from the free surface elevation at the time of the abrupt change in the slope of the unfiltered force time series up to the maximum free surface elevation. The change of location of the peak of the free surface elevation between two wave gauges was used to calculate the celerity of the wave which was used as the initial velocity of the wave slamming the cylinder. The resulting calculated model and data impulses are plotted on top of the force time series in figure 3 and figure 4 for cases 3 and 4 with azimuth limits of $\pi/4$ and $\pi/5$ respectively.



(a) (b)

Figure 3: The force time series with and without the slamming effect in addition to the data and model impulse, $\theta_{max} = \pi/4$, plotted as shades of gray and blue respectively for (a) case 3 and (b) case 4.



(a) (b)

Figure 4: The force time series with and without the slamming effect in addition to the data and model impulse, $\theta_{max} = \pi/5$, plotted as shades of gray and blue respectively for (a) case 3 and (b) case 4.

For all the cases the total impulse from the measurement is plotted against the model impulse for $\theta_{max} = \pi/4$ and $\theta_{max} = \pi/5$ in figure 5. It is seen that for the chosen events the choice of $\theta_{max} = \pi/5$ is more consistent with the measurements.

4 Discussion

The results present our first validation of the recent pressure-impulse model to experimental data. Several choices must be made to detect the slamming impulse from the measured inline force and free surface time histories. Further choices and uncertainty again are associated with the determination of the input parameters to the pressure-impulse model. The present approach, however, is found to show consistent results for this initial set of six events.

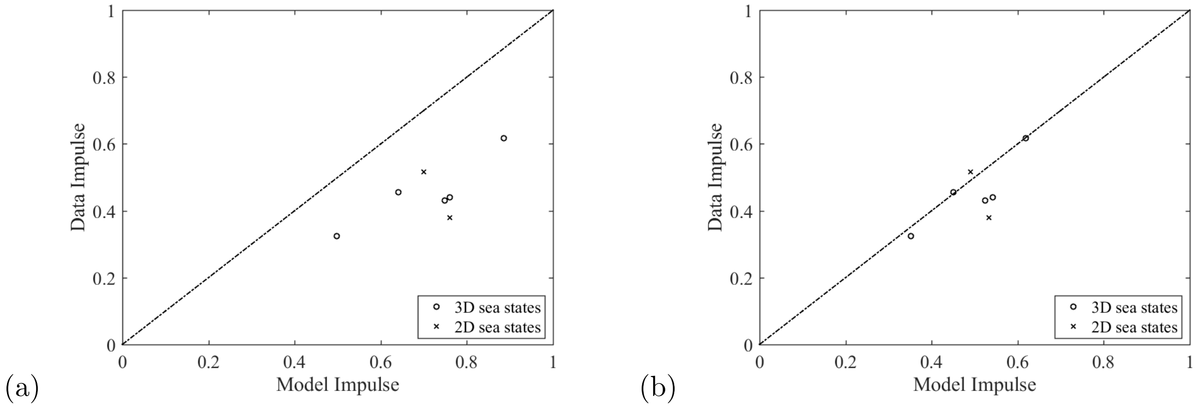


Figure 5: The data impulse versus model impulse for (a) $\theta_{max} = \pi/4$ and (b) $\theta_{max} = \pi/5$.

While an initial value for the maximum impact angle of $\theta_{max} = \pi/4$ was tested first, a better match for the value of $\pi/5$ was seen. Analysis of more events and further CFD results are planned to yield more insight into the best possible value. As slamming loads are in nature associated with strong variability, the ambition is to devise a central recommended value and next quantify the scatter around it. The pressure impulse formulation is well suited in this regard, since its integral form can be expected to be more robust than the resolved force peak height and duration. This gives potential for application in connection to fully nonlinear wave kinematics for improved design calculations.

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