A non-hydrostatic model for nonlinear water waves interacting with structures

Yuxiang Ma, Congfang Ai, Changfu Yuan, Guohai Dong

State Key Laboratory of Coastal and Offshore Engineering, Dalian University of Technology, Dalian, 116023, China
Email address of the present author: yuxma@dlut.edu.cn

HIGHLIGHT
A 3D non-hydrostatic model for water waves interacting with structures is developed by solving the Navier-Stokes equations with an explicit projection method. The immersed boundary (IB) method is incorporated in the model to deal with structure surfaces. The grid system is built based upon a horizontal Cartesian coordinate and a vertical boundary-fitted coordinate system. The capability of the present model is validated against several experimental data for waves interacting with different-types of structures.

1. INTRODUCTION
The so-called non-hydrostatic models for water waves are models that track the free surface motion using a single-valued function of the horizontal plane, which is derived by integrating the continuity equation and meanwhile applying kinematic boundary conditions at the free surface and the bottom. Even though non-hydrostatic models cannot resolve overturning free surface, they are computationally more efficient in contrast to VOF-based Navier-Stokes Equations models (Zijlema and Stelling, 2005; Ai et al., 2011). By incorporating the IB method, non-hydrostatic models in a σ-coordinate were developed to study wave structures interactions (Kang et al., 2015; Ma et al., 2016). However, when implementing the IB method in σ-coordinate system, a grid point inside the rigid structures at one time may move out at another time and vice versa, which can cause some errors. We present here a 3D non-hydrostatic model to solve wave-structure interactions by incorporating the direct-forcing IB method. The 3D grid system is built from a 2D horizontal structured mesh by adding a vertical general boundary-fitted coordinate system. This vertical grid system allows for a great adaptability of the vertical discretization and meanwhile maintains the boundary-fitted properties of better fitting the bed and free surface. By reasonably distributing horizontal grid layers, the issue of σ-coordinate system aforementioned can be avoided in the general boundary-fitted model.

2. NUMERICAL MODEL
The governing equations of the non-hydrostatic free surface flow consist of the 3D incompressible Navier-Stokes equations and the continuity equation. By splitting the pressure into hydrostatic and non-hydrostatic ones, $p = g(\eta\text{-}z) + q$, the governing equations can be obtained. The detailed description of the equations is illustrated clearly in Ai et al. (2011), therefore, they are not presented here for brevity.

The governing equations are solved in a 3D grid system, which is built from a 2D horizontal structured mesh with multiple horizontal layers. In such a grid system, the 3D grid projection on the horizontal plane forms a rectilinear grid system, which has a set of N_x and N_y cells in x and y directions, respectively. In the vertical direction, a general boundary-fitted coordinate system is employed and the physical domain is divided into N_z layers (see Fig. 1).

Taking i, j and k as the grid indexes in the x, y, and z directions, respectively. A 3D cell with center at (i, j, k) is bounded by the intersection of the water column between the bottom and the free surface with the horizontal levels $z_{k\pm1/2}$ and the horizontal grid lines $x_{k\pm1/2}$ and $y_{k\pm1/2}$. The horizontal levels are defined following a vertical general boundary-fitted coordinate system given below (see Fig. 1).

$$z_{k+1/2} = \begin{cases} z_f + (k-k_f)[\eta(x,y,t) - z_f]/(N_z - k_f) & k > k_f \\ z_f & k = k_f \\ -h(x,y) + k[z_f + h(x,y)]/k_f & k < k_f \end{cases} \tag{10}$$

where z_f and k_f are predefined fixed horizontal level and the layer index, respectively. It should be noted that the vertical grid system presented in Eq. (10) has the boundary-fitted properties. The levels above z_f may be function of time, while below it the levels are function of space and they are fixed because the bottom surface is immovable in this study. Therefore, by setting z_f above submerged structure, we can avoid interpolation errors induced by implementing IB method in a moving grid system (e.g., σ-coordinate).

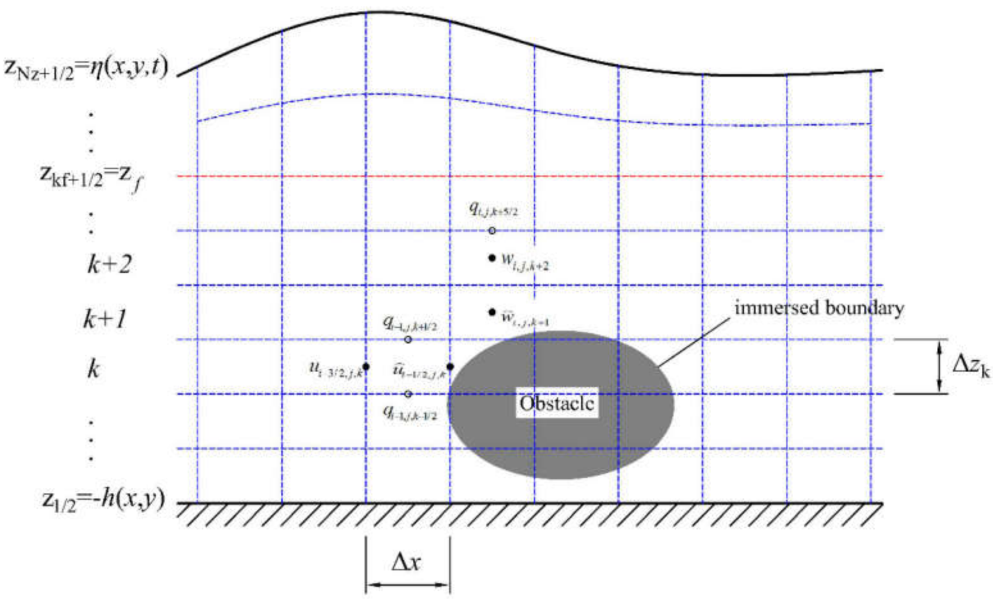

Figure. 1. Vertical general boundary-fitted coordinate system with an immersed boundary on the $x-z$ plane.

The 2D variables η and h are defined at the geometric center of the 2D horizontal mesh cells, which is denoted by (i, j). The horizontal velocities u and v are stored at the centers of the cell faces (i+1/2, j, k) and (i, j+1/2, k), while the vertical velocity w is stored at the cell centers (i, j, k). Finally, the non-hydrostatic pressure component is stored at the cell faces, which are denoted by (i, j, k+1/2).

The detailed discretization of the equations and the numerical algorithm can be referred to Ai et al. (2011) and Ai et al. (2018).

3. MODEL VALIDATIONS

The capabilities of the developed model are then evaluated by considering existing experimental data for wave interactions with different types of structures. For brevity, only two cases are presented here. As shown in Figure 2, the first is a case for a solitary wave interacts with a suspended horizontal plate (Lo and Liu, 2014). This test is more challenging for numerical models because of its complex flow field around structure. It is found that the present model predicts the wave profiles well by comparing with the experimental data. The vortices generation and evolution also can be resolved (Figure 3).

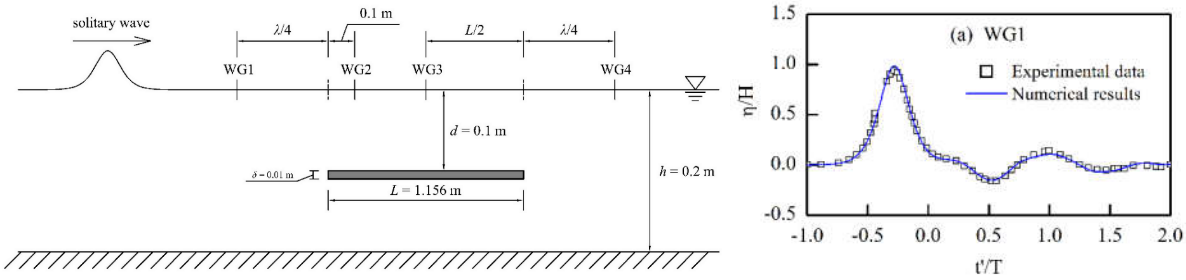

Figure 2. Schematic of solitary wave incident on a suspended horizontal plate (left); comparisons of free surface elevations between numerical results and experimental data for a case with height 0.04m and effective period 2.115s (right).

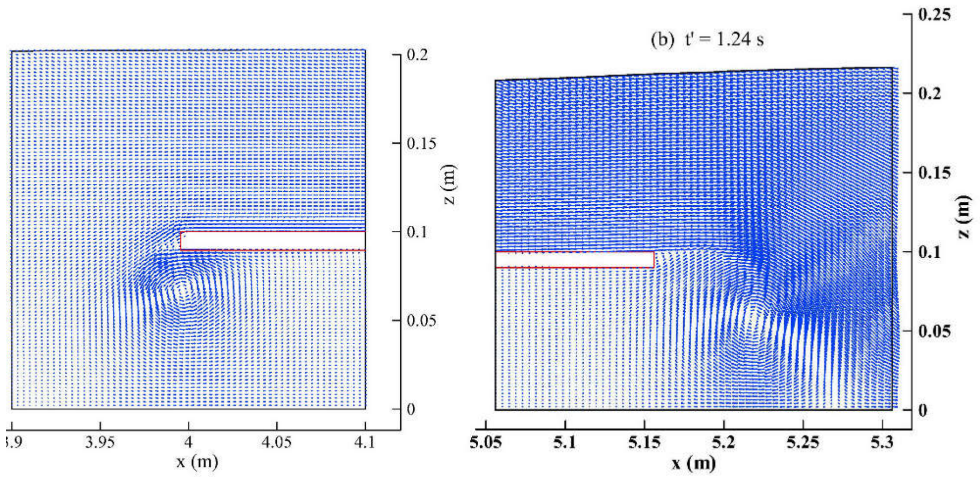

Figure. 3. Computed velocity fields near the left and the right edges of the plate for a case with height 0.04m and effective period 2.115s at t' = 1.24s (t' is relative to the time when the wave peak passed gauging point WG2.).

In order to test the capability of developed model for simulating wave interactions with 3D structures, simulations for a solitary wave scattering from a vertical circular cylinder (Figure 4) are presented. Figures. 5 and 6 show comparisons of dimensionless free surface elevations between numerical results and experimental data by Yates and Wang (1994) at two selecting gauging points along the radius lines $\theta = 0^\circ$ and 100°, respectively.

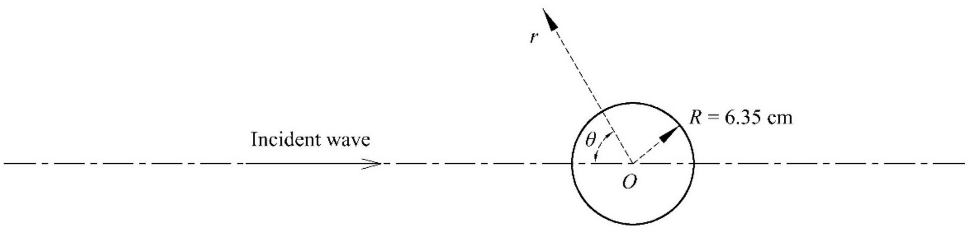

Figure. 4. Schematic of solitary wave scattering from a vertical circular cylinder.

A combination of the polar angle θ and the radial distance r/R measured from the cylinder center is used to identify the selected gauging points. Moreover, θ is measured in the clockwise direction and the $\theta = 0^\circ$ line represents the upstream centerline of the flume. The comparison results indicate that the model predictions agree well with the measured data.

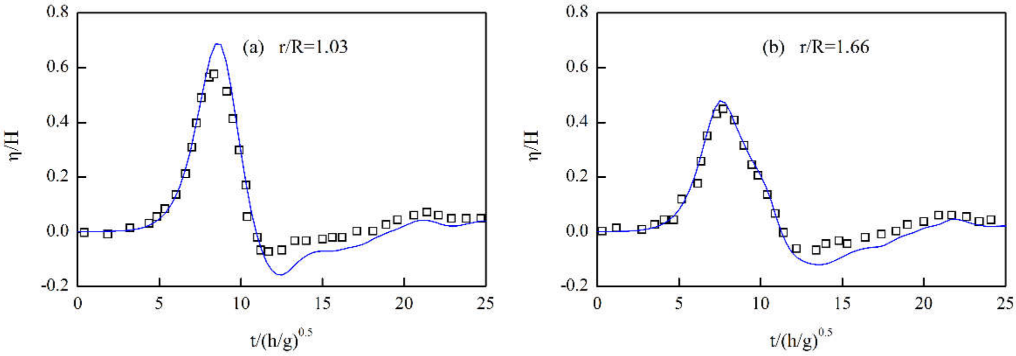

Figure 5. Comparisons of free surface elevations along $\theta = 0^o$ between numerical results(solid lines) and experimental data (open squares).

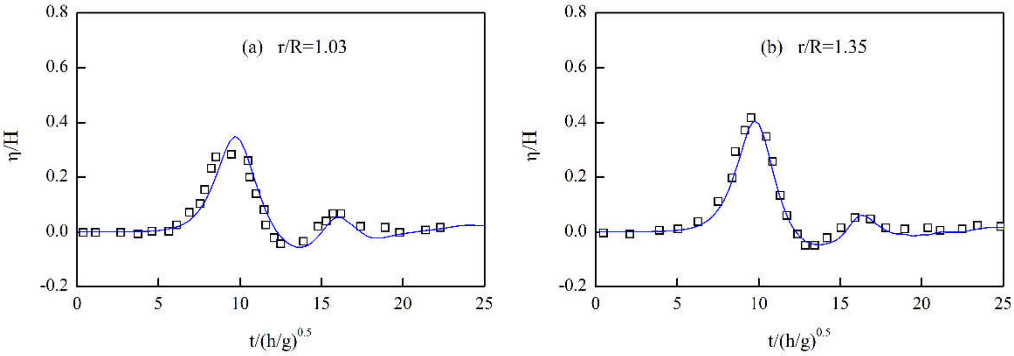

Figure 6. Comparisons of free surface elevations along $\theta = 100^o$ between numerical results (solid lines) and experimental data (open squares).

ACKNOWLEDGMENTS

The research was financially supported by the High-Tech Ship Research Projects Sponsored by the Ministry of Industry and Information Technology (MIIT) of China (Grant No. 2016-23-7).

REFERENCES

Ai C, Jin S, Lv B. A new fully non - hydrostatic 3D free surface flow model for water wave motions. Int J Numer Meth Fluids, 2011, 66(11):1354-70.

Ai, C.F., Ma, Y.X. Yuan, C.F., Dong, G.H., Semi-implicit non-hydrostatic model for 2D nonlinear wave interaction with a floating/suspended structure, Euopean Journal of Mechanics, B/Fluids, 2018, 72: 545-560.

Kang A, Lin P, Lee YJ, Zhu B. Numerical simulation of wave interaction with vertical circular cylinders of different submergences using immersed boundary method. Comput Fluids, 2015,106:41-53.

Lo H-Y, Liu PL-F. Solitary waves incident on a submerged horizontal plate. J Waterway, Port, Coastal, Ocean Eng 2013;140(3):1-17.

Ma G, Farahani AA, Kirby JT, Shi F. Modeling wave-structure interactions by an immersed boundary method in a σ-coordinate model. Ocean Engineering, 2016, 125:238-47.

Yates GT, Wang K. Solitary wave scattering by a vertical cylinder: experimental study. In: Proc 4th Int Offshore Polar Eng. Conf. 1994.

Zijlema M, Stelling GS. Further experiences with computing non - hydrostatic free - surface flows involving water waves. Int J Numer Meth Fluids, 2005, 48(2):169-97.