

A fast method for optimization of large wave energy converter arrays

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Introduction

Wave energy converter (WEC) arrays consist of devices extracting energy from their oscillatory motion. We define large WEC arrays as those consisting of $N = O(100)$ devices. Through favorable wave interaction, the extracted energy by a N -body WEC array can be significantly larger than that extracted by the same number of isolated devices. This increase in extraction performance is quantified by the array gain $q(k, \theta_I) = P(k, \theta_I)/(N P_0(k))$, where P is the power extracted by a N -body WEC array in monochromatic waves of wavenumber k incident at an angle θ_I , and P_0 is the power extracted by a single axisymmetric constituent body when operating in isolation in the same incident wave.

In addition to k and θ_I , the array gain q strongly depends on the spatial array configuration. A full diffraction and radiation formulation of the wave-body interactions in large WEC arrays is a computationally complex problem. As a result, optimization studies of WEC array configurations have been limited to arrays of small number of bodies ($N = O(10)$) [1, 2, 6]. A common approximation to allow for the calculation of q in larger arrays is to assume that the devices are so-called point absorbers, which do not scatter waves, so the overall wave field consists of incident and radiated waves only. This results in a much simpler calculation of q , and the optimization of much larger WEC arrays is computationally feasible [e.g. 3]. However, the point-absorber approximation is valid when the body size a is small ($ka \ll 1$), so when the scattering is strong ($ka = O(1)$), important physical phenomena are not modeled correctly [e.g., see 8].

Here we present a fast method for optimizing the performance of large WEC arrays with $O(100)$ bodies, taking into account the exact wave interaction hydrodynamics. The wave-body interaction is based on a novel formulation [7, 8] of the classic multiple scattering framework [4]. The gradient of the objective function required for gradient-based optimization algorithms is calculated using the fast adjoint method. We present results of spatial configuration optimization of rectangular arrays with up to 150 bodies and up to 53 optimization variables, with the objective of obtaining maximum array gain q . These represent the largest WEC arrays optimized to-date where the full diffraction and radiation problem is taken into account.

Mathematical model

We consider linear incident waves governed by potential flow and interacting with an array of oscillating bodies; we assume body motions are small. We express the total potential in terms of the incident wave and the scattered and radiated waves from each body, and expand these in terms of partial waves [5].

Within the novel formulation of multiple scattering theory [7, 8] the dependency of the resultant wavefield on the spatial array configuration is encoded in the separation matrix alone, a matrix whose components depend on the distance between bodies i and j only; all the body-related quantities are expressed through transfer matrices obtained for a body in isolation (diffraction transfer matrix, diffraction force transfer matrix, dynamics+radiation transfer matrix). The unknown amplitudes of the scattered and radiated partial waves are obtained from imposing the boundary conditions on each body (expressed through diffraction transfer matrix). This results in a global coupled linear system for the unknown complex scattered partial wave amplitudes \mathbf{c}

$$\left(\mathbf{I} - \mathbf{T}(\mathbf{S}(\mathcal{C}))^T \mathbf{H}\right) \mathbf{c} = \mathbf{T} \mathbf{d}(\mathcal{C}), \quad \text{or} \quad \mathbf{A}(\mathcal{C}) \cdot \mathbf{c} = \mathbf{b}(\mathcal{C}). \quad (1)$$

Here, \mathbf{I} , \mathbf{T} and \mathbf{H} are block-diagonal identity, diffraction transfer and radiation transfer matrices, respectively (each diagonal block corresponds to one body); \mathbf{S} is the global separation matrix dependent on the configuration \mathcal{C} (the superscript $(\cdot)^T$ denotes a matrix transpose); \mathbf{d} is the global block incident wave vector. The $M \times M$ complex system matrix \mathbf{A} and the $M \times 1$ complex vector \mathbf{b} depend on \mathcal{C} . The size of the system $M = N \times N_w$ depends on the number of bodies and the total number of partial waves N_w . The expression (1) is in principle exact (other than the truncation to N_w partial waves), i.e. the full diffraction problem is taken into account, including the effects of evanescent waves. It is valid for bodies of general shape (not necessarily all equal) that can be fixed ($\mathbf{H} \equiv \mathbf{I}$), freely oscillating or extracting energy. The extracted power by an array can be expressed as $P = \mathbf{c}^+ \mathbf{\Omega} \mathbf{c}$, where $\mathbf{\Omega}$ is a known block-diagonal power extraction matrix; q is calculated from P following its definition.

For the optimization problem, the configuration \mathcal{C} is parameterized by a vector of optimization variables $\boldsymbol{\chi}$, so the array gain can be expressed as $q(\boldsymbol{\chi}; \mathbf{c}(\boldsymbol{\chi}))$. The total gradient $dq/d\boldsymbol{\chi}$ of array gain q with respect to $\boldsymbol{\chi}$ can be expressed as

$$\frac{dq}{d\boldsymbol{\chi}} = \frac{\partial q}{\partial \boldsymbol{\chi}} + \frac{\partial q}{\partial \mathbf{c}} \frac{\partial \mathbf{c}}{\partial \boldsymbol{\chi}} + \frac{\partial \mathbf{c}^+}{\partial \boldsymbol{\chi}} \frac{\partial q}{\partial \mathbf{c}^+} = 2 \operatorname{Re} (\nabla_{\mathbf{c}} q \cdot \nabla_{\boldsymbol{\chi}} \mathbf{c}) = 2 \operatorname{Re} (\boldsymbol{\lambda}^T \cdot \mathbf{B}) \quad (2)$$

where $\nabla_{\mathbf{a}} g$ stands for a gradient of a general (scalar, vector or matrix) function g with respect to a vector \mathbf{a} . While $\nabla_{\mathbf{c}} q$ is easily found (the expression is analytically available), finding $\nabla_{\boldsymbol{\chi}} \mathbf{c} = \mathbf{A}^{-1} [\nabla_{\boldsymbol{\chi}} \mathbf{b} - (\nabla_{\boldsymbol{\chi}} \mathbf{A}) \cdot \mathbf{c}]$ is a very cumbersome process (requires solving $M \times M$ linear system N_p times). The entire trick of the adjoint method lies in parenthesizing differently the right hand side of (2) — instead of calculating and multiplying $\nabla_{\mathbf{c}} q \cdot \nabla_{\boldsymbol{\chi}} \mathbf{c}$ term by term, we can regroup the terms and recognize that the adjoint $\boldsymbol{\lambda}$ is the solution of an adjoint linear system $\mathbf{A}^T \cdot \boldsymbol{\lambda} = (\nabla_{\mathbf{c}} q)^T$. This $M \times M$ system is independent of the number of optimization variables N_p , and has the same complexity and characteristics as the original problem. This way, we only need to solve two $M \times M$ systems during each step of the optimization process, instead of $1 + N_p$ systems if we had done it in the straight-forward way. We term the matrix $\mathbf{B} = \nabla_{\boldsymbol{\chi}} \mathbf{b} - (\nabla_{\boldsymbol{\chi}} \mathbf{A}) \cdot \mathbf{c}$ in equation (2) the “adjoint Jacobian”. As the array goes through the changes in the spatial configuration during the optimization, only the separation matrix $\mathbf{S}(\mathcal{C})$ and the incident wave vector $\mathbf{d}(\mathcal{C})$ in (1) need to be updated; the body-related transfer matrices remain constant. Further details are given in Tokić [7].

Results

We optimize here spatial configurations of rectangular arrays for maximum gain q at a particular wavenumber k for a range of incident angles θ_I . The arrays consist of $N_x = 3$ rows of bodies in x -direction, with N_y bodies laid in y -direction in each row, totaling $N = N_x \times N_y$ bodies. The spacing between the rows (in x -direction) is denoted by c_1 and c_2 . The shifts in the y -direction of the second and third row relative to the first one (facing the incident wave) are s_1 and s_2 , respectively. The spacing d_i between the bodies in y -direction is the same in every row, and it is denoted by d_1, \dots, d_{N-1} . In total, there is $N_p = 2 \times (N_x - 1) + (N_y - 1)$ optimization variables ($\boldsymbol{\chi} = \{c_1, c_2, s_1, s_2, d_1, \dots, d_{N_y-1}\}$) parameterizing the array configuration. In particular, we optimize $N_x \times N_y = 3 \times 25$ and 3×50 arrays (75 and 150 bodies), resulting in 28 and 53 optimization variables, respectively. The arrays consist of truncated vertical cylinders of radius $a = 0.3h$ and draft $H = 0.3h$, where h is the uniform water depth. The power take-off rate is chosen such that at body resonant wavenumber k_r , an isolated body extracts maximum possible energy, i.e. capture width $W = 1/k_r$ [e.g. 5].

The array gain q is a highly multimodal function of k , θ_I and array configuration. In order not to get trapped in one of the local maxima that are significantly below the global optimum, the spatial configurations for every θ_I are initialized as uniform-spacing arrays that achieve maximum q . The initial uniform spacings c^0 and d^0 are obtained from a known $q(c, d)$ function for 3×20 arrays, Figure 1 [7].

The optimal gain q^* and corresponding optimal row spacings and shifts of 3×50 arrays for a range of incident angles are shown in Figure 2. The improvement over the initial gain q^0 is especially prominent

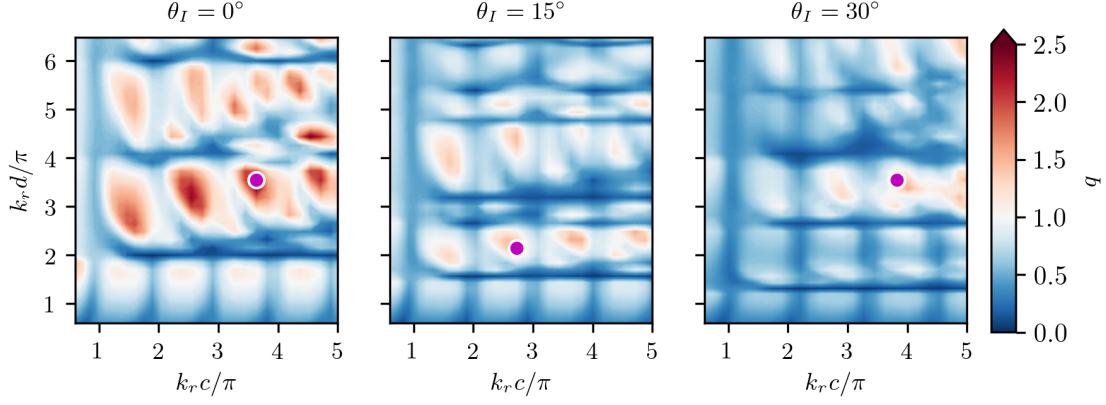


Figure 1: Array gain q of uniformly spaced 3×20 arrays as a function of spatial configuration (spacings c and d in x - and y -direction, respectively) at body-resonant wavenumber k_r and for three incident angles θ_I . The chosen initial configurations c^0 and d^0 are marked by magenta circles for each θ_I . The gain q for larger θ_I exhibits a similar character as that presented here, albeit with lower overall values.

for larger incident angles θ_I , where q^* is up to $\sim 30\%$ better than q^0 . The row spacings c_1^* and c_2^* do not deviate significantly from the initial values c^0 , and they remain almost identical to each other. The y -direction row shifts s_1^* and s_2^* become non-zero for non-zero incident angles, and a relationship between the row shifts $s_2^* \approx 2s_1^*$ is approximately valid. This indicates that the array configuration approximately forms a lattice. The angle γ of the row-wise lattice vector is $\gamma \approx \arctan \frac{s_1^*}{c^*}$, or $\gamma \approx \arctan \frac{d^* - s_1^*}{c^*}$, where the chosen expression is based on the first body in the back row that has equal or larger y -coordinate. Interestingly, angle γ of the lattice vector is closely proportional to θ_I , indicating that the bodies in the back rows of the optimal configurations are placed such that they are directly behind the one in the front row along the incident propagation angle, and not such that they are “out of the shadow” of the front row.

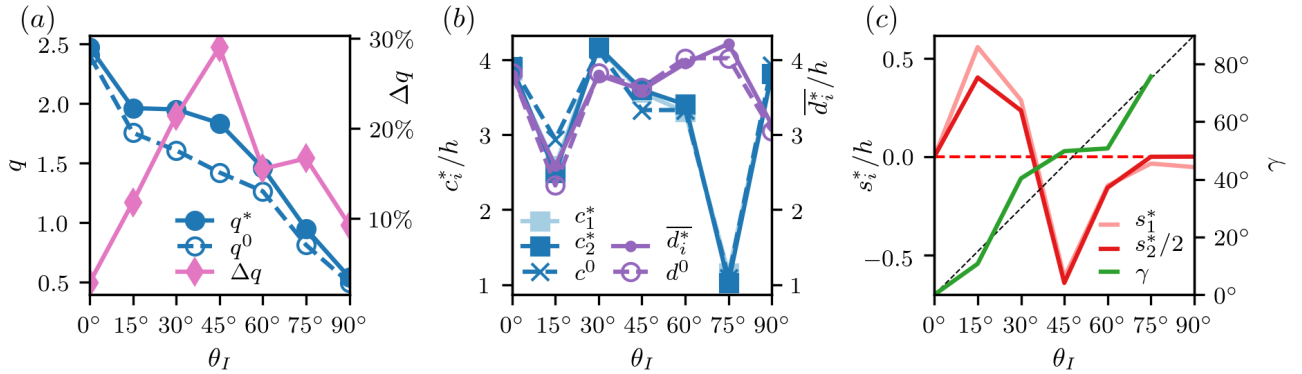


Figure 2: Optimal 3×50 rectangular array. (a) Optimal gain q^* , initial gain q^0 (left axis), increase in gain $\Delta q = (q^* - q^0)/q^0$ (right axis). (b) Optimal x -spacing c_i^* (left axis), the average of optimal y -spacings $\overline{d_i^*}$ (right axis). Initial values are plotted in dashed line. (c) Optimal y -shifts s_i^* (left axis). The shifts are initially zero (dashed red line). The angle γ of the row-wise lattice vector (right axis). The $\gamma = \theta_I$ (dashed black line) is added for reference.

The optimal spacings d_i^* along the y -direction for $N_y = 25$ and $N_y = 50$ arrays for three incident angles is shown in Figure 3. The initially uniform spacing d^0 is now oscillating between the bodies, with oscillations $\Delta d_i^* \equiv (d_i^* - \overline{d_i^*})/\overline{d_i^*}$ being larger at the edges of the array ($\overline{d_i^*}$ is the average of the optimal

spacings $\overline{d_i^*}$). For $N_y \gtrsim 25$, the number of bodies in a row N_y no longer affects the optimal spacing as the corresponding spacings (counted from the rows' ends) for $N_y = 25$ and $N_y = 50$ arrays are almost identical. For normal incidence, the optimal spacings d_i^* are symmetric with respect to the middle of the row. With the increase in incident angle, the asymmetry between optimal spacings at the two ends of the row grows. The optimal spacings d_i^* for incident angles $\theta_I > 30^\circ$ follow the same characteristics as presented here.

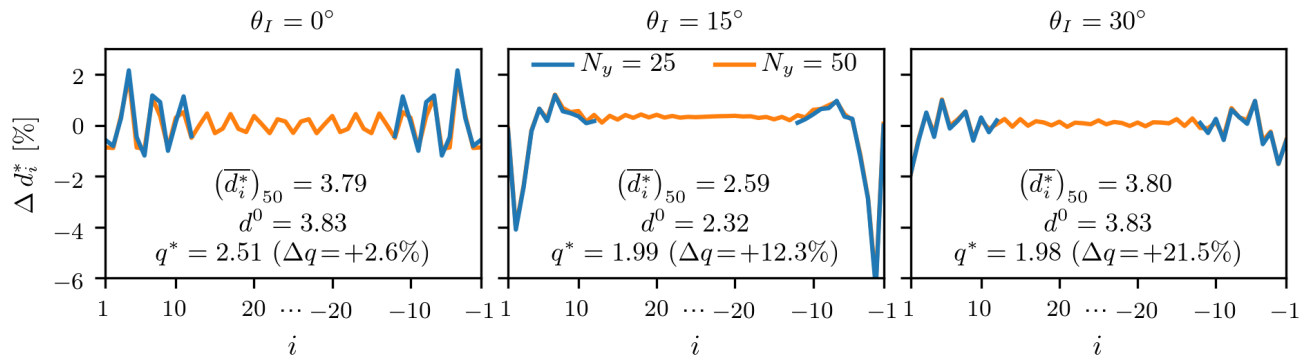


Figure 3: Optimal spacing d_i^* for 3×25 and 3×50 array for three different incident angles θ_I as a function of the spacing index i (spacing between bodies i and $i + 1$ in a row). The negative indices i refer to counting from the end of the row. The average optimal spacing $\overline{d_i^*}$ for the 3×50 array is stated for reference.

Discussion

The results presented here indicate that further improvements in array gain q are possible (up to $\sim 20\%$) if the spatial configuration of large WEC arrays is optimized. The presented method is applicable to general array configurations consisting bodies of general shape operating in general sea states, i.e. no assumption on the body size, the character of the diffraction problem, or on optimality of body motion is imposed. Together with its computational efficacy, this makes the presented method also well-suited for optimization of large arrays in broad-banded seas.

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