

# Jon: teacher and learner



Jon was always telling anecdotes (with purpose)  
always teaching,  
always learning, ...

- Born 20 May 1951. Grew up in St Andrews, Scotland.
- Did the 11plus "The pressure on my 11 year old self was horrendous, even though I was capable of doing well in the exam and did so"
- Wanted to be a historian, but decided on maths at the last minute
- BA (Hons Math) '71 Western Ontario
- MSc '72 and DPhil '74, Oxford, in functional analysis and optimisation
- '74-'93 Dalhousie (with stints at Carnegie-Melon, Waterloo, Limoges, ... )
- '93-'04 Simon Fraser University as Shrum Professor of Science
- '04-'09 Dalhousie as Canada Research Chair
- '09-'16 Newcastle University, Australia (with stints at Dalhousie, Western , ... )

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*'82 Computation and Experimental Maths*
- '93-'04 Simon Fraser University as Shrum Professor of Science  
*Centre for Experimental and Constructive Mathematics (CECM)*
- '04-'09 Dalhousie as Canada Research Chair  
*'95 Organic Maths Project*  
*Dalhousie Distributed Research Institute and Virtual Environment (D-Drive)*  
*Interdisciplinary Research in the Mathematical and Computational Sciences (IRMACS)*
- '09-'16 Newcastle University, Australia (with stints at Dalhousie, Western , ... )  
*Computer Assisted Research Mathematics and its Applications (CARMA)*

# Jon's words: why he chose maths over history

Somewhat unusually, I can exactly place the day at registration that I became a mathematician and I recall the reason why. I was about to deposit my punch cards in the 'honours history bin'. I remember thinking

*"If I do study history, in ten years I shall have forgotten how to use the calculus properly. If I take mathematics, I shall still be able to read competently about the War of 1812 or the Papal schism."* (Jonathan Borwein, 1968)

From *"Implications of Experimental Mathematics for the Philosophy of Mathematics"*

# Connected thinker - a typical 'Jon' talk slide

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## 24. Pi's Childhood

43. Pi's Adolescence

48. Adulthood of Pi

79. Pi in the Digital Age

113. Computing Individual Digits of  $\pi$

## Links and References

Babylon, Egypt and Israel

Archimedes Method circa 250 BCE

Precalculus Calculation Records

The Fairly Dark Ages

## Arithmetic was Hard

- See DHB & JMB, "Ancient Indian Square Roots: An Exercise in Forensic Paleo-Mathematics," *MAA Monthly*. 2012.
- The prior difficulty of arithmetic<sup>2</sup> is shown by 'college placement' advice to a wealthy 16C German merchant:

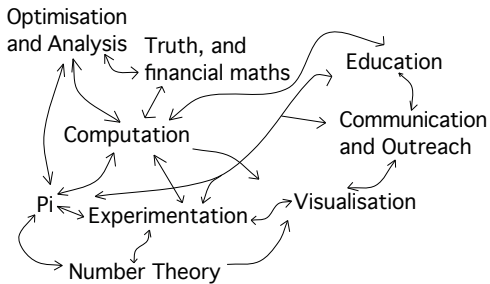
*If you only want him to be able to cope with addition and subtraction, then any French or German university will do. But if you are intent on your son going on to multiplication and division — assuming that he has sufficient gifts — then you will have to send him to Italy.*

— George Ifrah *or* Tobias Danzig

<sup>2</sup>Claude Shannon (1913-2006) had 'Throback 1' built to compute in Roman, at Bell Labs in 1953.



# Themes and Jon

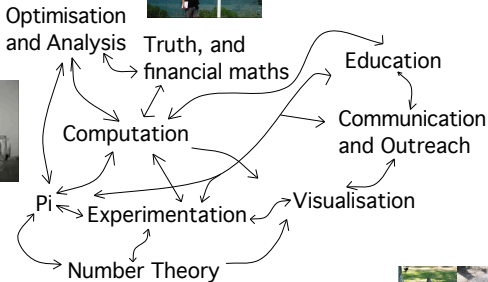


# Themes and Jon



Carving His Own Unique Niche, In Symbols and Stone

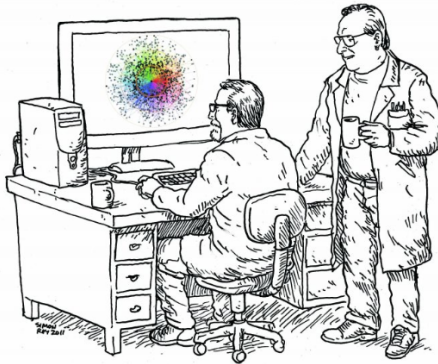
CMS D. Bonein Prize: Maching



- ▶ It is hard to keep up with a single research area in maths
- ▶ Jon was fluent and prolific across many!
  
- ▶ The following images are taken from Jon's talks and papers, ever illustrating connections ...



## Veselin and Jon:

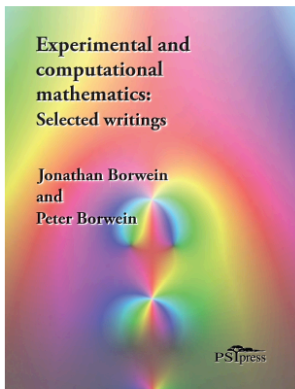


*"Sometimes it is easier to see than to say."*

4. CARMA's Mandate  
12. About CARMA  
18. My Current Interests  
30. Computing Individual Digits of  $\pi$

5. Experimental Mathematics  
10. CARMA's Mandate  
11. CARMA's Objectives  
12. Communication, Computation and Collaboration

## Communication and Computation: are entangled



### Communicating Mathematics (2008, 2010)

- See <http://carma.newcastle.edu.au/jon/c2c08.pdf> for chapter on Access Grid.



J.M. Borwein

CARMA and Me

# Pictures as Datasets



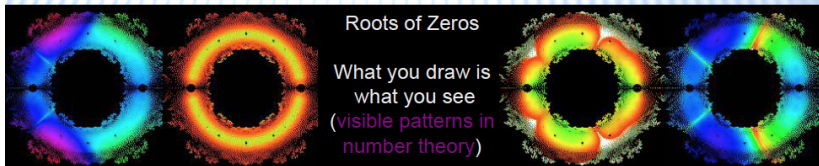
**Effective Teaching  
Effective Learning**  
in the Quantitative Disciplines

Striking fractal patterns formed by plotting complex zeros for all polynomials in powers of  $x$  with coefficients 1 and -1 to degree 18 (from [Organic Maths](#))

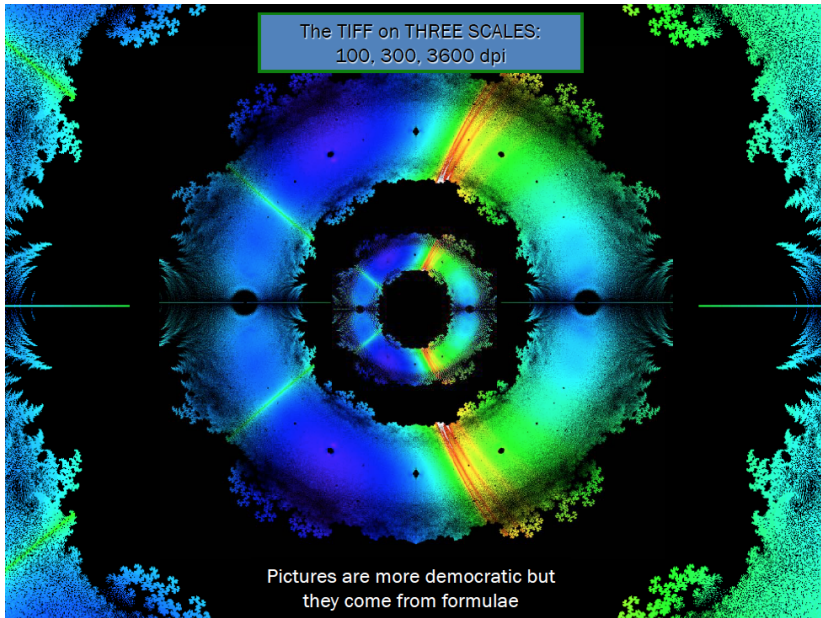
Coloration is by sensitivity of polynomials to slight variation around the values of the zeros. The color scale represents a normalized sensitivity to the range of values; red is insensitive to violet which is strongly sensitive.

- All zeros are pictured (at 3600 dpi)
- Figure 1b is colored by their local density
- Figure 1d shows sensitivity relative to the  $x^9$  term
- The white **and orange** striations are not understood

A wide variety of patterns and features become visible, leading researchers to totally unexpected mathematical results



The TIFF on THREE SCALES:  
100, 300, 3600 dpi



Pictures are more democratic but  
they come from formulae

# Research, Teaching & Outreach



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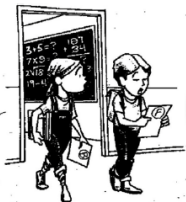
❖ Are each helped by the others - and by collaborative technology - but **content comes first**



# Technology Includes



**Effective Teaching  
Effective Learning**  
in the Quantitative Disciplines



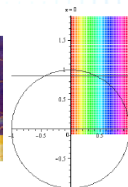
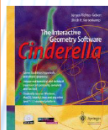
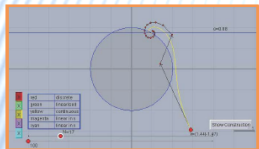
"It's not the math I hate...It's the aftermath."



**Tom Lehrer's**  
That's Mathematics



Politeness → Maturity → Sensitivity



## All Artefacts:

- ❖ Cartoons
- ❖ Pictures
- ❖ Films
- ❖ Music
- ❖ Animations
- ❖ Simulations
- ❖ Spreadsheets
- ❖ Packages
- ❖ Applets
- ❖ Haptics
- ❖ Virtual Reality
- ❖ Blogs (Math Drudge)
- ❖ JSTOR, Amazon, iTunes...

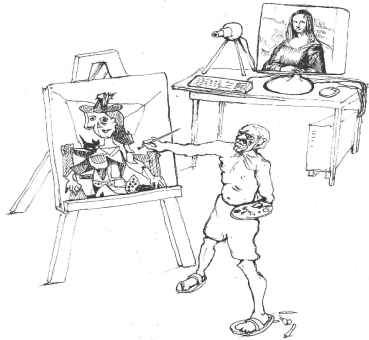
❖ <http://www.carma.newcastle.edu.au/~jb616/lm-june.htm>

# Jon & Veselin on technologies and “Organic Maths”



In the early 1990s a group of researchers, J. Borwein, P. Borwein, R. Corless, L. Jörgenson, and N. Sinclair, all then affiliated with the Center for Experimental and Constructive Mathematics at Simon Fraser University—as part of a National Telelearning Network—started the Organic Mathematics Project (OMP).<sup>1</sup> One of the main goals of the project was to achieve a more meaningful integration of the technologies then available, moving towards the ideal environment described as follows:

A mathematician working in ideal conditions would be able to look at a fresh problem and easily access any related material, find all the work on simpler but similar problems, and quickly carry out any subcomputations needed for the solution of the fresh problem. Such a person would also be able to consult freely not only with colleagues but with experts with whom he or she/was not previously familiar.



**“Computers are useless. They can only give you answers.”**

Jon drew conclusions about maths that apply to maths teaching,  
eg



Jon drew conclusions about maths that apply to maths teaching,  
eg

## Some early conclusions:

So I am sure they get made

**Key ideas:** randomness, normality of numbers, planar walks, and fractals

Maths can be done *experimentally* (it is fun)

- using computer algebra, numerical computation and graphics: SNaG
- computations, tables and pictures are experimental data
- but you can not stop thinking

Making mistakes is fine

- as long as you learn from them
- keep your eyes open (conquer fear)

You can not use what you do not know

- and what you know you can usually use
- you do not need to know much before you start research (as we shall see)



How not to experiment

# Jon also wrote explicitly about teaching, eg

In brief

and

at length

## Teaching is Hard



"Just a darn minute! — Yesterday you said that X equals two!"



Effective Teaching  
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in the Quantitative Disciplines

- Especially teaching easy things
- You can't take liberties
- Technology can help
- But only so much
- Unis can't afford much

PRMT 1: Visual Theorems      Digital Assistance      PRMT 2: Case Studies      Other References

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**Tools and Mathematics**      April 2016

J. Monaghan, L. Trouche, J.M. Borwein  
**Tools and Mathematics**  
Instruments for learning  
Series: Mathematics Education Library

- The only book on the topic of tools and mathematics education
- Comprehensive coverage from pre-history to future directions in the field
- Content divided equally among the areas of curriculum, assessment, and policy design

This book is an exploration of tools and mathematics and issues in mathematics education related to tool use. The book has four parts. The first part sets the scene with a reflection on doing a mathematical task with different tools, a mathematician's account of tool use in his work and historical considerations of tool use. The second part opens with a broad review of technology and intellectual trends, circa 1970, and continues with three case studies of approaches in mathematics education and the place of tools in these approaches. The third part considers issues related to mathematics instruction: curriculum, assessment and policy; the calculator debate; mathematics in the real world; and teachers' use of technology. The final part looks to the future and digital tools: task design; the importance of artifacts in gameplay; and new forms of activity via connectivity.

1st ed. 2016. XX, 481 p. 123 illus., 92 illus. in color.  
Printed book

Jonathan Borwein (University of Newcastle, Australia)      Visual Theorems      www.corms.newcastle.edu.au/vts

# Jon raised many questions

Quoting from Bailey and Borwein: "Exploratory Experimentation and Computation"

We turn to observations, many of which have been fleshed out in coauthored books such *Mathematics by Experiment* [9], and *Experimental Mathematics in Action* [3], where we have noted the changing nature of mathematical knowledge and in consequence ask questions such as "How do we teach what and why to students?", "How do we come to believe and trust pieces of mathematics?", and "Why do we wish to prove things?"

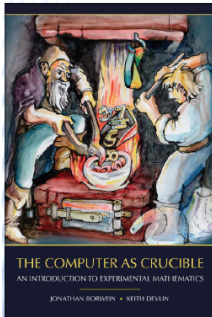
# Jon raised many questions

## Resources not Courses?



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In the Quantitative Disciplines

- ❖ Current and expected advances in mathematical computation and scientific visualization make it now possible to do (teach, learn) mathematics in many varied and flexible ways.
- ❖ We'll continue to explore and flag the opportunities to integrate computational, graphic and other tools into our teaching - for philosophic, pedagogic and aesthetic reasons



- ❖ <http://www.experimentalmath.info>
- ❖ <http://carma.newcastle.edu.au/portal/>
- ❖



# Collected resources

## My Maths Portal

### Jon Borwein's Mathematics Portal

The following is a list of useful math tools. The distinction between categories is somewhat arbitrary.

#### Utilities (General)

1. [The On-Line Encyclopedia of Integer Sequences](#)
2. [ISC2.0: The Inverse Symbolic Calculator](#)
3. [3D Function Grapher](#)
4. [Julia and Mandelbrot Set Explorer](#)
5. [The KnotPlot Site](#)
6. [The Cinderella Geometry Site](#)

#### Utilities (Special)

7. [RRP Digit Database](#)
8. [Integer Relations Interface: PSLO and LLL](#)
9. [EZ Face: Evaluation of Euler Sums and Multiple Zeta Values](#)
10. [GraPHedron: Automated and Computer-Assisted Conjectures in Graph Theory](#)
11. [ProofWeb a system for teaching logic and for using proof assistants through the web](#)
12. [Embrex-Trefethen-Wright Pseudospectra and Eigenproblems](#)
13. [Symbolic and Numeric Convex Analysis Tools](#)

#### Reference

14. [NIST Digital Library of Mathematical Functions](#)
15. [Experimental Mathematics Website](#)
16. [Numbers, Constants, and Computation](#)
17. [Numbers: the Competition](#)
18. [The Prime Pages](#)
19. [MathResource Online Dictionary](#)

#### Content

20. [Math in the Media \(from the AMS\)](#)
21. [Wolfram Mathworld](#)
22. [Planet Math](#)
23. [Wikipedia: Mathematics](#)
24. [Euclid in Java](#)
25. [Finch's Mathematical Constants](#)

#### Math Courses 2011

26. [Multi Zeta Values Honours Course 2010](#) (Borwein and Zudilin)  
Given over the [AMSI Access Grid Network](#)



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Coxeter 1930

CARMA 2010



# Lived his ideas



## 2010: Introduction to Experimental Mathematics



AMSI Honours Course



**Experimental Mathematics** is the use of a computer to run computations - sometimes no more than trial-and-error tests - to look for patterns, to identify particular numbers and sequences, to gather evidence in support of specific mathematical assertions, assertions that may themselves arise by computational means, including search. Like contemporary chemists - and before them the alchemists of old - who mix various substances together in a crucible and heat them to a high temperature to see what would happen, today's experimental mathematician puts a hopefully potent mix of numbers, formulas, and algorithms into a computer in the hope that something of interest emerges.

This course will provide a rigorous introduction to Experimental Mathematics while also exploring a variety of pure and applied mathematical topics that the student may well not have seen during an undergraduate degree.

**Course Goals** The successful student will emerge from this course with much enhanced abilities to:

- Use current mathematical computation tools
- Learn mathematics independently
- Formulate and refine conjectures
- Develop strategies for proof or refutation of new mathematics

• **Resources and Links** These will grow over time.

- **Maple**
  - Maple ini file
  - Aug 18 Maple session (2009)
  - Sept 1 Maple session (2009)
- AMSI **2010 Courses**
- My **Mathematics Portal**
- **Experimental Mathematics** website and **Math Drudge** blog

## From a joint book review Jon and I wrote:

**JO:** What about the claim that mathematics should be used as a filter to enter professions that don't need it, as described in *Loving and Hating*?

**JB:** We filter inappropriately. We conflate the idea of the ability to do mathematics well with the ability to appreciate it. We don't make the same mistake with Shakespeare, or sports.

What I would like to be able to do, when teaching mathematics, is to ask students in the class who is there for appreciation, and who needs mastery for subsequent professional use. Those in the first group could take the course as Pass/Fail, and those in the second for a numerical grade. I don't mind which group individuals are in. Both are worthwhile. With modern technology there is more and more capacity to meet both kinds of needs in the same classroom.

*What do we think?*



## Panel Questions:

In light of Jon's work on the changing nature of mathematics: tools, spaces, experimentation, and in view of the character of your own research areas, **can we:**

1. Bring research practices fruitfully into school maths?
2. Teach maths so that ordinary people don't fear it?
3. Change research training to be more inclusive?  
and
4. How are tools and inter-disciplinarity changing *us*?