Number Theory Down Under
24-25 October, 2014

L/Prof. Jon Borwein (CARMA)

Title. Computation and structure of character polylogarithms with applications to character Mordell-Tornheim-Witten sums.

Abstract. This lecture and the associated paper provides tool to study character polylogarithms. These objects are used to compute Mordell-Tornheim-Witten character sums and to explore their connections with multiple-zeta values (MZVs) and with their character analogues.
This lecture is based on joint work with David Bailey in an eponymous paper

Tim Trudgian (ANU)

Title. A trigonometric inequality related to the zeta-function.

Abstract. The size of the zero-free region of the Riemann zeta-function is connected with the distribution of primes. I shall talk about a trigonometric problem, going back to Landau, the solution to which provides a slightly better zero-free region. This is joint work with Mike Mossinghoff.

Adrian Dudek (ANU)

Title. On the Riemann Hypothesis and the Difference Between Consecutive Primes.

Abstract. This talk will give an introduction to the theorems one can deduce regarding the distribution of prime numbers if the Riemann Hypothesis is assumed. Moreover, the speaker will discuss his recent results in this area.

Dzmitry Badziahin (Durham)

Title. On continued fraction expansion of van der Poorten's Laurent series and Thue–Morse constant.

Abstract. Let t be a Thue-Morse constant, i.e. the real number whose binary expansion is comprised by the infinite Thue-Morse word. In 2003 Allouche and Shallit asked wether t is badly approximable. The study of this questions leads us to the formal Laurent series mentioned by van der Poorten in several of his works: $f(Z) = \prod_{i=0}^{\infty}(1 - Z^{-2^i})$. We manage to find the explicit formulae for computing the continued fraction expansion of slightly modified series $f(Z)/Z$. This allows us to answer negatively to Allouche and Shallit question.

Alex Ghitza (Melbourne)

Title. Irreducibility of Hecke polynomials in higher levels.

Abstract. A popular way of specifying a modular form is by giving its Fourier expansion. As this is an infinite power series, for practical purposes we are interested in questions such as: given two eigen forms, how many Fourier coefficients do we have to compare in order to distinguish the forms? I will describe some theoretical and computational results, and give a simple relation between this question and the properties of the Hecke polynomials of spaces of modular forms. Most of the work is joint with Sam Chow (Bristol) and James Withers (Boston University).

Angus McAndrew (Melbourne)

Title. Theta Operators and Galois Representations for Siegel Modular Forms.

Abstract. One of the guiding goals of the theory of modular forms is to understand their position in the Langlands conjectures. That is, the correspondence between modular forms and Galois representations. Though this is well developed in the elliptic case, the Siegel case remains quite mysterious. In the case of positive characteristic, the theta operator plays a crucial role for elliptic forms. I will describe some recent work investigating how this is mirrored for Siegel modular forms.

Yohei Tachiya (Hirosaki)

Title. Linear independence of Lambert series associated with various sequences.

Abstract. (This is a joint work with Florian Luca) We investigate linear independence over $\mathbb{Q}$ of Lambert series associated with various sequences. For example, let $\{a_n\}_{n\geq 1}$ be an eventually periodic sequence of rational numbers, not identically zero. Then the value of $\sum_{n=1}^{\infty} a_n z^n/(1-z^n)$ at $z = 1/q$ is irrational, where q is a rational integer with $|q| > 1$. This generalizes a result of Erdős who treated the case $a_n = 1$ $(n \geq 1)$.

As an application of our theorem, we can deduce linear independence over $\mathbb{Q}$ of the numbers

$$1, \quad \sum_{n=1}^{\infty} \frac{1}{q^n - 1}, \quad \sum_{n=1}^{\infty} \frac{1}{q^{2n} - 1}, \quad \ldots, \quad \sum_{n=1}^{\infty} \frac{1}{q^{\ell n} - 1}$$

for any integer $\ell \geq 1$.

Wadim Zudilin (Newcastle)

Title. Every MZV has two sides.

Abstract. Multiple zeta values (MZVs) possess a rich algebraic structure of algebraic relations, which is conjecturally determined by two different (shuffle and tuffle) products of a certain algebra of noncommutative words. In a very recent work, Henrik Bachmann constructed a q-analogue of the MZVs for which the two products are dual to each other, in a very natural way. In my talk, I plan to overview the construction of Bachmann, its links to MZVs, some progress on understanding the two products as well as some related (and open) linear independence problems.

Igor Shparlinski (UNSW)

Title. Distribution of Points on Varieties over Finite Fields.

Abstract. We give a survey of recent results on the distribution of rational points on algebraic varieties that belong to a given box. Generally, these results fall into two categories:

- large boxes, where asymptotic results are possible;
- small boxes, where only upper bounds are possible.

We discuss results of both types and also indicate the underlying methods. Some of these work for very general varieties, some apply only to very specific varieties such as hypersurfaces $x_1^{-1} + ... + x_n^{-1} = a$ that appear in the Erdos-Graham problem or "hyperbolas" $x_1 x_2 ... x_n = a$.

Besides intrinsic interest, the aforementioned results have a wide scope of applications in number theory and beyond. These include: bounds of Kloosterman sums, shifted power identity testing, distribution of elliptic curves in isogeny and isomorphism classes, polynomial dynamical systems in finite fields and several others.

Stephen Meagher (UNSW)

Title. Low genus curves over finite fields and Theorems of Deligne and Howe.

Abstract. The theory of Honda and Tate describes isogeny classes of Abelian varieties over a finite field. When the varieties in question are ordinary, a Theorem of Deligne beefs up the theory of Honda-Tate into a description of the category of ordinary abelian varieties over that field. The category in question turns out to be a certain category of free Z modules equipped with an operator (which does the work of Frobenius). Howe, in his thesis, explained how to describe polarisations in Deligne?s category. In this talk we will explain this and give some applications to proving the existence of low genus curves over finite fields with a specified number of points. A draw back of the method is that it does not give equations for the curves. The object of the talk is expository.

Andreas-Stephen Elsenhas (Sydney)

Title. Cubic surfaces - moduli spaces and arithmetic.

Abstract. By using normal forms and invariants several descriptions of the moduli space of cubic surfaces have been worked out in the 19th century. We give an overview of this. Finally we will apply these results to construct surfaces with prescribed geometric or arithmetic properties.

Simon Kristensen (Aarhus)

Title. On the error sum functions.

Abstract. The error sum functions $\mathcal{E}^*$ and $\mathcal{E}$ are defined for real numbers α by the formulas

$$\mathcal{E}^*(\alpha) = \sum_{m\geq 0} q_m\alpha - p_m, \quad \mathcal{E}(\alpha) = \sum_{m\geq 0} |q_m\alpha - p_m|,$$

where p_m/q_m is the sequence of convergents (or best approximations) of α. Error sum functions have relations to problems in both transcendence and the theory of q-series, but they are of interest in their own right. In joint work in progress with Carsten Elsner, we study the fractal structure of the graphs of the functions $\mathcal{E}^*(\alpha)$ and $\mathcal{E}(\alpha)$.

Min-Sha (UNSW)

Title. Pseudolinearly dependent points on elliptic curves.

Abstract. Given a subgroup Γ of rational points on an elliptic curve E over the rational numbers and a real $x \geq 2$, we call a rational point Q of E an x-pseudolinearly dependent point of Γ if it is not in the group Γ but belongs to the reduction of Γ modulo every prime $p \leq x$ of good reduction. In this talk I will construct such a point Q and present some unconditional and conditional upper bounds on its canonical height when the rank of Γ is less than that of E. This is joint work with Igor Shparlinski.

Judy-anne Osborn (Newcastle)

Title. Normal numbers and Dyck paths.

Abstract. We look at an open number-theoretic problem and see if insight can be gained by expressing it in terms of classic objects from enumerative combinatorics.

Eloise Hamilton (ANU)

Title. Bounding the nth prime explicitly.

Abstract. The prime number theorem tells us that the nth prime is asymptotic to $n \log n$. There are many applications in number theory for an explicit version of this. I shall outline the first such explicit method, given by Rosser, as well as some subsequent results and links to the Riemann zeta function.

David Harvey (UNSW)

Title. Counting points on smooth plane quartics.

Abstract. I will discuss progress on point-counting algorithms for smooth plane quartics over finite fields, i.e. nonhyperelliptic curves of genus 3. The goal is to provide data for investigating questions such as Sato-Tate for these curves. This is joint work with Andrew Sutherland (MIT).

Norman Wildberger (UNSW)

Title. A number theory conjecture from triangle chromogeometry.

Abstract. Rational trigonometry allows us to develop geometry completely algebraically, in particular Euclidean geometry can be studied over the rational numbers and finite fields, and relativistic geometries too. It turns out there is an intimate three-fold symmetry between Euclidean (blue) geometry and two relativistic geometries (red and green), and that the simultaneous existence of Incenters for a triangle in all three is a rather delicate issue, leading to some interesting conjectures.

Chul-Hee Lee (UQ)

Title. Nahm's conjecture and related algebraic structures

Abstract. Finding a criterion of when a q-hypergeometric series can have modularity is an interesting open problem in number theory. Nahm's conjecture relates this question to the Bloch group in algebraic K-theory. I will give an introduction to the conjecture and explain its close relationship with various objects such as the dilogarithm function, Y-systems and Q-systems.

Richard Brent (ANU and Newcastle)

Title. Twin primes (seem to be) more random than primes

Abstract. Cramer's probabilistic model (1936) gives a useful way of predicting the distribution of primes. However, the model is not perfect, as shown by Cramer himself (1920), Maier (1985), and Pintz (2007). With appropriate modifications (Hardy and Littlewood), Cramer's model can also be used for twin primes. In fact, computational evidence suggests that the model is better for twin primes than for primes. In this sense, twin primes appear to behave more randomly than primes. This can be explained by the connection between the Riemann zeta function and the primes - the zeros of the zeta function constrain the behaviour of the primes, but there does not appear to be any analogous constraint on the behaviour of twin primes. In the talk I will outline some of the results mentioned above, and present some numerical evidence for the claim that twin primes are more random than primes.

Michael Coons (Newcastle)

Title.

Abstract.