

AMSI/AustMS Workshop on Mathematical Thinking

14—16 November, 2018

Venue: X803, NewSpace (City Campus), The University of Newcastle Part of the CARMA Special Semester on Mathematical Thinking https://carma.newcastle.edu.au/meetings/mathematicalthinking/

| | Wednesday 14 November | Thursday 15 November | Friday 16 November |
|--|---|--|--|
| 08h00 | Registration | | |
| 08h20 | | | |
| 08h40 | Welcome | | |
| 09h00 | Keith Devlin | Kaye Stacey | Rafael Núñez |
| 09h20 | | Maura Sellars | |
| 09h40 | | | |
| 10h00 | Joel Pearson | Lynelle Campbell | James Franklin |
| 10h20 | | | |
| 10h40 | | Fran Baker | Теа |
| 11h00 | Теа | Теа | Peter Ellerton |
| 11h20 | George Willis | Eunice Chan + Robert Corless | |
| 11h40 | | | |
| 12h00 | Paul Garret | Asim Ghous | Brailey Sims |
| 12h20 | | | Richard Brent |
| 12h40 | Discussion | Discussion | Discussion |
| 13h00 | Lunch | WIMSIG Lunch | Lunch |
| 13h20 | | | |
| | | | |
| 13h40 | | | |
| 13h40 14h00 | Mathematics and Movement | | Veselin Jungic |
| 13h40 14h00 14h20 | Mathematics and Movement Session (no experience required): | | Veselin Jungic |
| 13h40 14h00 14h20 14h40 | Mathematics and Movement Session (no experience required): Bachman + Stern. Venue: Concert | Julia Collins | Veselin Jungic |
| 13h40 14h00 14h20 14h40 15h00 | Mathematics and Movement Session (no experience required): Bachman + Stern. Venue: Concert Hall | Julia Collins Michael Assis | Veselin Jungic Stephan Chalup |
| 13h40 14h00 14h20 14h40 15h00 15h20 | Mathematics and Movement Session (no experience required): Bachman + Stern. Venue: Concert Hall | Julia Collins Michael Assis | Veselin Jungic Stephan Chalup |
| 13h40 14h00 14h20 14h40 15h00 15h20 15h40 | Mathematics and Movement Session (no experience required): Bachman + Stern. Venue: Concert Hall Tea | Julia Collins Michael Assis Tea | Veselin Jungic Stephan Chalup Tea |
| 13h40 14h00 14h20 14h40 15h00 15h20 15h40 16h00 | Mathematics and Movement Session (no experience required): Bachman + Stern. Venue: Concert Hall <u>Tea</u> Rachel Bachman + Erik Stern | Julia Collins Michael Assis Tea | Veselin Jungic Stephan Chalup Tea Discussion |
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Titles and Abstracts

Michael Assis (University of Melbourne) Mathematical thinking in origami

Mathematics pervades many aspects of origami, from artistic design to engineering applications. I will give a broad overview of mathematical aspects of origami and then give personal examples of the use of mathematical thinking in the design of origami models.

Rachel Bachman and Erik Stern (Weber State University) Mathematics & Movement Workshop

In this participatory workshop, suitable and accessible for non-experienced movers, participants experience the relationship between kinesthetic/choreographic exploration of patterns and mathematical perceptions of pattern. The creative tasks embedded in the workshop will serve to launch discussions about the value of connecting artistic and mathematical thinking, and what constitutes interdisciplinary education and scholarship.

Rachel Bachman and Erik Stern (Weber State University) Mathematics & Movement – How and Why it Works

In this lecture, Rachel Bachman and Erik Stern report on eye-opening qualitative and quantitative assessment research comparing students who explored patterns through movement and mathematics to students from a conventional mathematics classroom. They will connect this work to research from the fields of mathematics education, arts integration, learning theory, and cognitive psychology. Also, the lecture will give attendees a look behind the curtain of how lessons are constructed, the key features that make these lessons so effective, and ways to adapt these methods for use in any mathematics classroom.

Rachel Bachman and Erik Stern (Weber State University) Embodied Learning: Connecting Movement & Mathematics (Public Lecture)

People think in a variety of ways – symbolically, interpersonally, kinesthetically, and more. Unfortunately, many classrooms do not provide access to a range of learning opportunities. Contrary to how subjects are often taught, it's natural to link modes of learning in order to gain rich understanding of a certain problem or subject. In this accessible and interactive presentation, the audience will join in a long-standing method of linking "modes" of thought to gain mathematical – and artistic – understanding. Don't worry – movement will be approached in an accessible and enjoyable way. See how mathematics can invite successful discovery for all students when the question is changed and multiple ways of knowing are encouraged.

Frank Baker (UON) Title TBA

Richard Brent (MSI and CARMA, ANU and UON) Thinking and Doing Mathematics in the 21st Century

We consider aspects of thinking and doing mathematics in the early 21st century, and the contrast with the early 20th century. In particular, we comment on the influence of computers and the Internet since the second half of the 20th century. To illustrate, we give a case study from the speaker's own work.

Lynell Campbell (Lynz Education) Unlocking the enigma that is mathematics and cracking the code through number sense

As surprising as it may seem, Maths is a subject that is not highly favoured or even valued by many in society. In fact, up to 25% of people in Western societies will self-identify as being maths anxious and 9% of people admit to being maths phobic. These unfortunate figures are reflected in classrooms, as there is increasing evidence that the way the subject is taught and assessed has contributed to this anxiety. While there are a number of research-based ways that classroom teachers can unlock the enigma of mathematics for students and reduce the perplexity around it, in this workshop, we'll consider how the explicit teaching and learning of number sense and the quantitative, comparative language 'experts' use to describe numbers can 'crack the code' for students in a primary mathematics classroom. We will look at the difficulties experienced by profoundly deaf students, who arrive at school with no language and are learning AUSLAN as their first language, in learning the language of maths and thereby developing 'number sense' in comparison to the difficulties experienced by students in mainstream schools.

Stephan Chalup (CARMA, UON) **Information Representation in Neural Networks**

The ability of artificial neural networks to process information similarly as neural networks in biology has been proposed for a long time. However, due to computational limitations the simulations of neural circuits have remained very restricted. With the availability of GPU computing and large scale artificial neural networks new opportunities to investigate information processing in larger neural systems have arrived. This talk will discuss some examples of information representation in artificial neural systems. The aim is to analyse activity in artificial neural networks not only for modelling results of biological neuro- and brain science but to increase our understanding of the mathematical structure of thought in neural systems in general.

Eunice Y. S. Chan and Robert M. Corless (University of Western Ontario) Computational Discovery on Jupyter

We are developing open education electronic resource materials (i.e., an OER digital textbook) for an active learning course in Computational Discovery. This talk is a progress report on this project. Once developed, the electronic materials could be used by any instructor skilled in active learning of computational mathematics. The course is modelled on highly successful courses previously offered at Western. The elements inherited from those courses, namely peer assessment of student projects, active learning, student-driven syllabus, and connection with modern research

tools, allow the students to actually do mathematics (as opposed to merely listening to and reading polished presentations of how we know some mathematics is true).

One of the key tools in achieving that ambitious goal is the use of visualization by computer. That, and the opportunities for interactive exploration and experimentation that are afforded by tools like Maple, Matlab, and now Python via Jupyter. The topics that we have chosen include continued fractions, fractals and Julia sets, Bohemian matrices, Iterated Function Systems, and the Chaos Game Representation for DNA sequences.

Julia Collins (CHOOSEMATHS, AMSI) Knitting Mathematics

There is a wonderful synergy between knitting and mathematics. Knitting is an inherently mathematical craft, using concepts from geometry, topology, number theory and coding. It can also be used as a medium to communicate mathematical ideas and visualise interesting shapes and patterns. In this talk I will highlight some of my favourite examples of the connections between maths and knitting, and will speak about the aspirations of my organisation Maths Craft Australia.

Keith Devlin (Stanford University) Title TBA

Peter Ellerton (University of Queensland) Critical thinking and mathematics

Critical thinking is ubiquitous as a stated education outcome regardless of discipline or institution. The relationship between critical thinking and mathematics, like most disciplines, however, is problematic. Do mathematicians and students of mathematics develop critical thinking skills by virtue of the fact that they do mathematics? In other words, is an education in mathematics sufficient for critical thinking? In terms of how the vast majority of people engage with mathematics, including most undergraduate students, I suggest that the answer is a resounding "no!" But this is surely not the case for professional mathematicians. Or is it? Understanding what we mean by critical thinking and mathematical thinking is a necessary condition for answering these questions. This talk is an attempt to do both so that the relationship between the two becomes clearer.

James Franklin (UNSW) Aristotelian realist philosophy of mathematics: what mathematics is really about

Mathematicians and those seeking to understand mathematical thinking have generally found little to help them in the kind of philosophy of mathematics that philosophers do. Standard Platonist and nominalist philosophies of mathematics have failed to address questions that outsiders have, such as, what is mathematics about?

We start again, with a philosophy of mathematics based on applied mathematics. Aristotelian (that is, non-Platonist) realism claims that mathematics is a science of certain aspects of the real world, just as much as physics or ornithology are. It studies quantitative aspects (such as ratio, numerosity, size) and structural aspects (such as symmetry, continuity, network topology). Mathematics is not

just a language, or manipulation of symbols, or logic, or an invention of our brains, but a straightforward science of our world (and other possible worlds).

Having established what mathematics is aiming to know, it can be inquired how we know it. Animals and infants can perceive, in the ordinary way, small numerosity, ratio and pattern. Humans can in addition understand why some such truths must be as they are. Chaining such insights creates proofs.

Paul Garret (School of Psychology, UON) Estimating Multiple Item-Sets: More Difficult Than You Think!

Like many species, humans can perform non-verbal estimates of quantity through our innate approximate number system. However, the cognitive mechanisms that govern how we compare these estimates are not well understood. Little research has addressed how the human estimation-system evaluates multiple quantities, and fewer studies have considered the cost to cognitive workload when undertaking these tasks. Here, we apply the mathematical tools of Systems Factorial Technology to a comparative estimation task. Across a series of experiments, we assess whether quantities i.e., non-overlapping red and blue discs, are estimated simultaneously (in parallel) or sequentially (in serial), and under what restrictions to cognitive workload. Our findings reveal that two item-sets may be estimated simultaneously through a parallel estimation system, under severe restrictions to cognitive workload capacity. These restrictions to workload extend to comparisons made with the subitizing range (item sets <4). While the estimation of a single item-set may be colloquially considered an effortless process, our results show that the estimation of multiple item-sets is a rather demanding feat.

Asim Ghous (Maplesoft Inc.) Using Digital Tools to Make Mathematical Thinking And Learning Fun

Students today are accustomed to the immediacy that comes with technology. Communication and information are constantly at their fingertips and they expect the same in the classroom. They expect learning that captivates them and holds their attention. In order to meet these expectations, educators must understand the motivations behind learning and find the proper tools to best facilitate the process.

There are many reasons people desire to learn, including success, meeting expectations and finding a successful career. Those who study because they are interested in the materials and want to learn seem to be in the minority. As instructors, how can you promote learning that is engaging and fun for all? Is it possible to have every student enjoy the learning process?

Modern digital learning tools, like DigitalEd's online suite of tools for online STEM education, Möbius, make it possible for educators to develop engaging and interactive content that offers a more hands-on learning experience and facilitates a deeper level of comprehension. Lessons can include text, videos, interactive applications, adjustable plots and more. In this presentation, discover how you can use digital tools to make learning fun and engaging so that all students enjoy the material and develop a strong desire to think and learn.

Veselin Jungic (Simon Fraser University) Why do I think of Jonathan Borwein whenever I hear the words "mathematical thinking"?

In this presentation, I will give several examples that illustrate the dynamic relationship between visualization and mathematical thinking. These examples will range from the beer glass puzzle to a classic theorem from Ramsey theory. I will also revisit some of Jonathan Borwein's work and ideas about computation, experimental mathematics, and mathematical thinking.

Rafael Núñez (UC San Diego) Towards a Science of the Nature and Foundations of Mathematics in the 21st Century

Mathematics is about abstract concepts, precise idealizations, relations, calculations, and notations, all of which are made possible by the amazing (albeit limited) workings of the human mind and the biological apparatus that supports it. Over the past 50 years the scientific study of mental phenomena has made enormous progress in understanding their psychological, linguistic, and neurological underpinnings. Traditional approaches to the questions of the nature and the foundations of Mathematics, made primarily in Philosophy (e.g., Platonism, Formalism, Logicism, Intuitionism, etc.) — developed many decades, if not centuries prior to these scientific developments— could not benefit from these findings. I argue that today, in the 21st century, questions such as What is mathematics? What is it for? How does it work? How to teach in a mind-friendly way? etc.— should be informed by, and be compatible with findings in the sciences of the mind. I'll illustrate my arguments with research addressing issues in hyperset theory and infinitesimal calculus, among others.

Joel Pearson (UNSW) The Mind wide shut: imagery and aphantasia

Maura Sellars (UON) Milestones, Mindsets and Math: Mapping Mathematical Meaning

Infants are born with mathematical capacities. How these are nurtured and developed depends on the interactions and environments that they experience in these early years and, later at school. This presentation focusses on the ways the neural pathways can be more readily accessed to develop the cognitive capacities of mathematizing, connecting and argumentation for preschool children. It draws on current literature and practice to illustrate the importance of positive disposition and explores pedagogies that allow students to develop inductive, deductive, abductive and adaptive reasoning skills from early in their school careers, whilst acknowledging the cognitive skills that serve as prerequisites for further development of the big mathematical ideas across all areas of the curriculum.

Brailey Sims (UON) Title TBA

Kaye Stacey (University of Melbourne) Teaching Mathematics as Applied Mathematical Problem Solving

Some mathematics teaching is routine, just as an exercise from a textbook for which you have received instruction will probably be routine. On other occasions, however, teaching mathematics is mathematically challenging, presenting problems of teaching for which the solutions may not be readily apparent. These situations require the application of mathematical knowledge in concert with the other types of knowledge required for teaching. In this sense, teaching mathematics is like applied mathematics - applying mathematical knowledge to the resolution of problems in another field. It involves the complex interplay of mathematical and teaching knowledge and processes of problem solving, with success judged according to how well students learn. The presentation explores this view of mathematics teaching through an examination of several teaching incidents.

George Willis (UON) The Creativity of Mathematical Thinking

Mathematics is based on intuitions gained from our everyday experience. However, the ideas built on that foundation take us well beyond everyday experience in a way that is profoundly creative and literally mind-expanding. Mathematical marvels from algebra and geometry will be highlighted in an attempt to frame the question: `Where does mathematics come from and where is it going?'



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NewSpace



City Hall

