Tools and Mathematics:
Instruments for learning
29 November–1 December, 2016

Room ELI 122

Sydney Campus, The University of Newcastle

CARMA

Contents

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- Naomi Borwein
- John Monaghan
- Judy-anne Osborn
- Malcolm Roberts
- Luc Trouche

Visit the workshop website at `https://carma.newcastle.edu.au/meetings/tools/`

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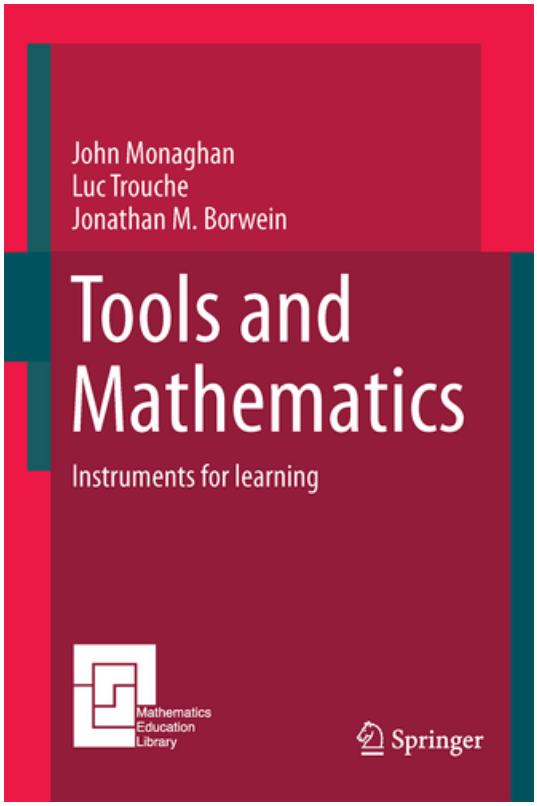
About the Book

"Tools and Mathematics" is an exploration of tools and mathematics and issues in mathematics education related to tool use. It comprises five parts: the first part reflects on doing a mathematical task with different tools, followed by a mathematician's account of tool use in his work. The second considers prehistory and history: tools in the development from ape to human; tools and mathematics in the ancient world; tools for calculating; and tools in mathematics instruction. The third part opens with a broad review of technology and intellectual trends, circa 1970, and continues with three case studies of approaches in mathematics education and the place of tools in these approaches. The fourth part considers issues related to mathematics instructions: curriculum, assessment and policy; the calculator debate; mathematics in the real world; and teachers' use of technology. The final part looks to the future: task and tool design and new forms of activity via connectivity and computer games. Because the tools created and used in mathematics (from ruler and compass to computer-enabled visualisation) have such a large impact on both mathematical thinking both in research and education, we hope the workshops theme will be a common point of interest that will enable many further such creative collaborations.

Authors

- Prof. John Monaghan, The University of Leeds and The University of Agder
- Prof. Luc Trouche, Ecole Normale Suprieure de Lyon
- Laureate Prof. Jonathan Borwein, CARMA, The University of Newcastle




Workshop
Dinner
55 Elizabeth Street
Vapiano
6 min
550 m
King St
George St
Pitt St
Castlereagh St
Elizabeth St
Phillip St
York St
Clarence St
Martin Pl
Barrack St
Martin Place Station
St James Station
MLC Centre
The Westin Sydney
Cenotaph
Hotel CBD
Bavarian Bier Café
The Strand Arcade
Telstra Sydney George Street
St James Church
King Street Courts


Workshop
55 Elizabeth Street
Central Station
Hill Park
The Rocks
Millers Point
Manly Fast Ferry
Circular Quay
Mrs Macquarie's Chair
Barangaroo
Mr. Wong
Bridge St
Royal Botanic Gardens
Wynyard
Ivy Complex
King Street Wharf
Apple Sydney
SEA LIFE Sydney Aquarium
St James
St Mary's Cathedral
Queen Victoria Building
Sydney Town Hall
Town Hall
Hyde Park
Darling Harbour
The Metro Theatre
Chinese Garden of Friendship
Museum
Kings Cross
Powerhouse Museum
The Soda Factory
Sydney's Paddy's Market
UTS Library
University of Technology Sydney
The Winery
Ann St
Albion St
Central
Chippendale
Surry Hills
UNSW Art & Design
Victoria Barracks
Moore Park
Pitt St
George St
Elizabeth St
College St
Crown St
Riley St
Bourke St
Oxford St
Eastern Distributor
Goulburn St
Burton St
Harris St
Lee St
Kippax St
Cowper Wharf Rd
Mrs Macquaries Rd
St Marys Rd
Forbes St
Victoria St
Broughton St
Phillip St
Bligh St
Murray St
M1

	TUE 29 NOV	WED 30 NOV	THU 1 DEC
10:00 am	**Naomi Borwein** ***"From Lipschitz to Homo Habilis Mathematicus: a case study of Jon Borwein"***	**John Monaghan** ***"Tool and mathematics - tools matter"***	**Luc Trouche** ***"From Scribal to Digital Schools, an Inspiring Journey in Mathematics (Education)"***
11:00 am	Morning tea		
11:30 am	Anthony Morphett *"Using visual blocks to support mathematical syntax"*	Jana Visnovska *"Designing for tool use in mathematics classrooms"*	Matt Skerritt *"Tools for Teaching Computational Mathematics"*
12:00 noon	Scott Lindstrom *"Phase Plotting in Differential Geometry"*	David Sherwood *"Round Table on Calculators"*	Paul Abbott *"Sums, Products, and the Zeta Function: Visualizing a $1,000,000 Problem"*
12:30 pm	Lunch		
2:00 pm	**Michael Assis** ***"Origami Mathematics in Education"***	**Michael Barnsley** ***"The tools of fractal geometry, 1978-2016"***	**Uri Wilenski** ***Phenomena-based and Generative modeling – integration of computational math and science***
3:00 pm	Bill Blyth *"Applied optimization problems solved using algebra and approximation in the small (algebraic no calculus) with visualisation enabled by CAS (Maple)"*	Bill Blyth *"eTeaching and eAssessment of Vector Calculus"*	MESIG AGM
3:30 pm	Afternoon tea		
4:00 pm	Tristram Alexander *"The computer as a tool to develop mathematical thinking"*	Sitti Patahuddin *"Understanding Mathematics Teachers TPACK through the Examination of Critical Instructional Events"*	
4:30 pm	Ljiljana Brankovic *"Gamification of STEM courses: What Works and How to Do it"*	Discussion panel 4:30 pm until 6:00 pm	
5:00 pm			
6:00 pm			
7:00 pm		Dinner 6:30 for 7:00 pm Venue: Vapiano, corner King and York streets	

Discussion Panel

"Changing the way people think, move and feel mathematically: the contribution of digital technologies"

The workshop incorporates a discussion panel with Paul Drijvers, Celia Hoyles, Ulrich Kortenkamp, Richard Noss and Judy-anne Osborn. This will take place on Wednesday , 30th November, from 4:30 to 6:00 pm, chaired by Elena Prieto (CARMA, Australia) and Celia Hoyles (Institute of Education, London).

Panel members:

- Paul Drijvers, Freudenthal Institute, Utrecht University, Netherlands

  Paul Drijvers is professor in mathematics education at the Freudenthal Institute of Utrecht Universitys Science Faculty. His research interests include the role of ICT in teaching, learning and assessing mathematics, algebra education, and teacher education. Paul has a background in preservice and in-service teacher training and published numerous articles and books on mathematics education. He also works as a part-time research scientist at Cito, the leading Dutch testing and assessment institute. More information is available at http://www.uu.nl/staff/PHMDrijvers.

- Celia Hoyles, Institute of Education, London, UK

  Prof Dame Celia Hoyles is Professor of Mathematics Education at University College London Institute of Education. She was the UK Government Chief Adviser for mathematics, 2004-07, the first recipient of the International Commission of Mathematics Instruction (ICMI) Hans Freudenthal medal in 2004 and the Royal Society Kavli Education Medal in 2011, both for research in mathematics education. One enduring research interest has been the design of computer environments to engage learners of all ages with mathematics.

- Ulrich Kortenkamp, Department of Mathematics, University of Potsdam, Germany

  Ulrich Kortenkamp is Professor of Mathematics Education at the University of Potsdam in Germany. He is working mainly at the intersection of Mathematics, Computer Science and Mathematics Education, trying to find ways to teach mathematics with and without ICT at all ages. Recent projects include research in teaching with digital tools in primary education and fostering creativity through digital tools in secondary education. He is external member of CARMA and serves as head of the department "Online Resources and Digital Tools" of the German Center for Teacher Education in Mathematics, DZLM.

- Richard Noss, Institute of Education, London, UK

  Richard Noss is Professor of Mathematics Education at the UCL Institute of Education. He was the founding director of the London Knowledge Lab, director of the government-funded Technology Enhanced Learning Research Programme, and deputy scientific manager of the EU Kaleidoscope network of excellence. Richard holds a Masters degree in pure mathematics and a PhD in mathematical education. He is an invited Fellow of the Institute of Mathematics and its Applications, a Fellow of the Academy of the Social Sciences and is a foreign fellow and medallist of the Union of Bulgarian Mathematicians. Richard was editor-in-chief of the International Journal of Computers for Mathematical Learning.

- Judy-anne Osborn, CARMA, The University of Newcastle, Australia

  Judy-anne Osborn is a mathematician interested in education, employed as a researcher and lecturer at the University of Newcastle. Judy-anne did her undergraduate (first class hons) and

PhD at the University of Melbourne, where she joined ARC Centre of Excellence "Mathematics and Statistics of Complex Systems". Dr Osborn then joined the Mathematical Sciences Institute at ANU as a postdoctoral fellow, followed by a stint as a postdoctoral fellow at Newcastle in the University's Priority Centre CARMA (Computer Assisted Research Mathematics and its Applications) before taking up her present position. Judy-anne has a long-term interest in the notion of mathematics as an exploratory game. This notion influences both her research in combinatorics and her teaching. Dr Osborn has had a number of publications in well-regarded mathematical journals as well as several teaching-related grants, including leading the Newcastle node of a current OLT grant to improve the training of mathematics teachers.

Many decades ago, Seymour Papert rose to the challenge of developing a theory and an appropriate languagesuch as objects-to-think-with, body syntonicity and aesthetic engagementthat could describe the way in which the use of digital technology changes the way people think, move and feel mathematically. Since then, the emergence of new digital technologies and new theories have helped researchers recognise the breadth and depth of that change and simultaneously provide a framework for the design and implementation of computational tools for learning mathematics.

The possibility of putting mathematical objects into motion, for example, fundamentally changes the nature of these objects, how they are perceived and reasoned about; moving these objects changes the bodily actions and gestures of both learners and teachers; making the objects transform, collide and overlap changes the stories that can be told about them. Research on the use of digital technology has also provided an extraordinary "window" on mathematical meaning making, to use the metaphor provided by Celia Hoyles and Richard Noss, in part because of the visibility of thought, motion and feeling enabled in expressive digital technology environments. Like Papert, Jonathan Borwein recognised that the task is essentially epistemological not only psychological nothing less than the construction of a new, computationally based mathematics. He often argued that computational mathematics is a "more honest" form of mathematics in that it "does not hide or obscure the experimental process by which a mathematical hypothesis is discovered".

In the light of the above views, this panel will address the new ways people think, move and feel mathematically, thanks to the opportunities offered by digital technologies.

Tools and Mathematics Abstracts

Abstracts are listed alphabetically by author's surname.

Sums, Products, and the Zeta Function: Visualizing a $1,000,000 Problem

A/Prof. Paul Abbott

In 1865 Lewis Carroll, the Oxford mathematician (1855-81), wrote Alice's Adventures in Wonderland. A colleague took the liberty of updating possibly the most famous opening lines in English literature as follows: "What is the use of a book without pictures, conversations, or experiments?"

This question is the focus here. A concrete example of interactive experimentation applied to sums, products, and the Riemann zeta function, will be used to provide an answer.

The treatment is informal, the focus is on visualisation, and the intention is broader teaching outreach.

The computer as a tool to develop mathematical thinking

Dr Tristram Alexander

In this talk I will discuss some of the pitfalls, and some of the successes, I have encountered in using a computer to develop mathematical thinking in first year engineering students and third year mathematics students. Across two courses the computer was used to: explore different ways of approaching a problem; uncover patterns and generate hypotheses; develop and practice algorithmic thinking; confirm or explore expected results; and engage in inquiry-based learning on research-level problems. I will provide some examples of the approaches used for these various activities. The practice of algorithmic thinking was found to be particularly challenging for students. Overall, student engagement was found to be significantly shaped by student past experience with the tool, and this needed to be accounted for in the teaching and learning environment.

Origami Mathematics in Education

Michael Assis

Origami has a long history in education, starting in 1835 with the kindergarten movement in Germany by Friedrich Froebel. Since the 1980s, the field of origami mathematics has grown considerably, as origami artists have sought mathematical tools to assist them in their designs and mathematicians have sought to apply their tools to bear on various facets of folding. In the last decade, a couple of classroom textbooks have been published compiling many mathematical topics, applicable from primary to university level classes. There are many connections which can be made to mathematics, depending on which aspects of origami one focuses on, from differential geometry to group theory. In this talk we'll cover several connections, including circle packings, trisecting angles, polynomial root finding, as well as coloring problems on the square lattice.

The tools of fractal geometry, 1978-2016

Prof. Michael Barnsley

Tools (specialist software, magnifying glasses, rulers, digital cameras) play a key role in the development of fractal geometry. Via experiments, typically involving pictures and algorithms, conjectures are made and tested, and theorems are proved. A famous example concerned early work on the the connectivity of the Mandelbrot set. Also applications are developed, for example, fractal image compression. In this lecture I will describe what seem now to be the most important tools I have used in fractal geometry, and the mathematics they have led to.

Applied optimization problems solved using algebra and approximation in the small (algebraic – no calculus) with visualisation enabled by CAS (Maple)

Prof. Bill Blyth

After following an experimental maths approach in research (in finding all possible exact solutions of nonlinear integral equations) I have used a similar approach in innovative teaching (the topic of this presentation): introducing solution of applied optimization problems pre-calculus - using algebra and approximation in the small (algebraic - no calculus) with visualisation enabled by CAS (Maple). We focus on problems with cubic polynomial objective functions and mention how this pre-calculus approach can be extended to other classes of objective functions.

From Lipschitz to Homo Habilis Mathematicus: a case study of Jon Borwein

Naomi Borwein

This talk surveys a spectrum of Jonathan M Borwein's expository writing on mathematics, creativity, and experimentation in academic and popular presses. From visualisation and intelligent computing as research tools, to comparative analysis of primary school math problems that go viral, these pieces reveal a serious and passionate engagement with mathematical education and culture, and a seemingly playful interaction with popular science and society, through intelligent design debates (ergo modern homo habilis mathematicus), mathematical history and philosophy (the humorous ontogeny of Pi is not equal to 22/7), the politics of financial maths (scientific fraud), and on. Indeed, Borwein's interdisciplinary approach is a guiding apparatus or paradigm for the trajectory of his research and didactics, where 'accessible examples' and 'experimental approaches' become his sharpest tools.

Gamification of STEM courses: What Works and How to Do it

Prof. Ljiljana Brankovic, Stephan Chalup, Prabhu Manyem, Matt Skerritt and Joel Wong

Gamification refers to the use of elements of games in non-game contexts and has been applied in workplace, marketing, health programs and other areas, with mounting evidence of increased interest, involvement, satisfaction and performance of the participants. More recently gamification has been emerging as a teaching method that has a great potential to improve students' motivation and engagement. Gamification in education should not be confused with playing educational games, as it only uses concepts such as points, leader boards, etc, rather than computer games themselves. In this talk we describe the gamification of a theoretical computer science course we performed in 2014/2015 as well as our experience with two other STEM courses.

Tool and mathematics – tools matter

John Monaghan

I will start my talk with a simple task, bisecting an angle, and argue the case that the tool we use to execute the task matters from the points of views of the mathematics involved in executing the task and what a learner may glean from the task. This will lead to a consideration of the import of Jonathan M Borwein's work in mathematics and the use of computers in school mathematics.

Understanding Mathematics Teachers' TPACK through the Examination of Critical Instructional Events

Dr Sitti Patahuddin, Thomas Lowrie, Barney Dalgarno

Technological pedagogical and content knowledge (TPACK) is a well-known framework for understanding the professional knowledge required by teachers to effectively teach specific content with technology. It has been used in mathematics education research and shown to be useful in designing mathematics professional development programs or courses for pre and in-service teachers. Professional development on TPACK should allow mathematics teachers to utilise technology better in their teaching practices. However, it was found that the enacted pedagogies did not always align to the pedagogies emphasized during the TPACK professional development. Thus, examination of classroom practices is required to understand the TPACK framework and the complexity of technology integration in mathematics classrooms. This case study seeks to explore a series of critical instructional events using TPACK to address the question: What TPACK constructs are most influential in shaping and understanding mathematics teachers pedagogical practices using digital technology? This case study was conducted in an early secondary mathematics classroom in Indonesia that used a web-based resource to support students understanding of fractions. The finding suggests that the qualitative examination of the four intersected TPACK constructs (PCK, TCK, TPK, and TPACK) assists in understanding the challenges and the opportunities to teachers when utilising an exploratory based-technology. It demonstrates that the combination of pedagogical stances and choice of technology significantly influence the visibility of other TPACK constructs. This study implies the need of thoughtful planning prior to using web-based resources and the importance to utilise critical events in developing and assessing teachers TPACK. It also suggests that examining critical events of authentic teaching can provide the opportunity to gain better insights into what types of teacher knowledge is needed to effectively integrate technology into mathematics teaching. Teachers can draw upon the TPACK framework as a guideline to ensure their planning accommodates for the possible challenges that might emerge during the learning process. The examination of critical events using the TPACK framework might complement traditional questionnaire-based approaches for understanding the complexity, opportunities and challenges of the integration of technology into mathematics teaching. Therefore, TPACK researchers might consider, for instance, assessing teachers TPACK knowledge through critical events scenario-based interviews.

Round Table on Calculators

David Sherwood

This will be a round table on developments in the capabilities of scientific calculators and their effect on the teaching of tertiary (and transition to tertiary) level mathematics and the setting of assessment tasks.

From Scribal to Digital Schools, an Inspiring Journey in Mathematics (Education)

Luc Trouche

My talk starts with evidence of a complex system of artefacts supporting mathematical practices (teaching and learning) 4,000 years ago in scribal schools in Mesopotamia. I will then fast forward to the present and teacher "orchestrations" in mathematics classrooms, situations where a variety of tools are made available and each student develops these tools into mathematical instruments. This, I hold, is the raison d'être of schools for learning mathematics and I will provide several examples to support this assertion.

Designing for tool use in mathematics classrooms

Dr Jana Visnovska, Jose Luis Cortina

In our contribution, we explore some of the issues of tool use in mathematics education. We are specifically interested in these issues as design researchers who develop means of supporting the mathematical learning in classrooms. We view our work as entailing the development of both supports for the teachers work of planning and teaching, and resources for teacher professional development.

We will use examples from our work on fractions as measures sequence to note two emerging connections to discussions in Tools and Mathematics book. First, we are interested in purposes that tools and representations (in use) serve in students mathematical work, and how these purposes change over time. We are aware that our understandings ofand often intuitions aboutthese purposes shape our choices and design of artefacts (e.g., stick, straws, and scissors vs. computer tools visualising similar measuring actions), which then shape what kinds of insights learners get to develop.

Second, we are finding that learning how other researchers design and use artefacts in teaching and in researching learning, even within the same mathematical domain, is non-trivial. The artefacts are not transparent, academic papers afford limited insight, and communicating across differing conceptualizations of learning and how it can be supported brings its own complications.

We approach the conference as an opportunity to learn more within the outlined spaces.

Phenomena-based and Generative modeling – integration of computational math and science

Uri Wilensky

MESIG Abstracts

eTeaching and eAssessment of Vector Calculus

Prof. Bill Blyth

We discuss our Vector Calculus course taught entirely in Maple with assignments marked automatically with Maple where Marking Reports and a cumulative Marks List are generated. My CAA marking procedure allows graphs, comments etc also to be marked by a tutor in a very efficient way and the marks with comments are inserted into the Marking Reports and the Marks List.

Phase Plotting in Differential Geometry

Scott Lindstrom, Paul Vrbik

We extend the notion of phase plotting in complex analysis to its natural analog in differential geometry via conformal mappings. While phase plotting on Riemann spheres with stereographic projection has been done previously, we explore hyperbolic motions with pseudospheres as canvas.

We discuss some ideas for possible employment of these tools within the context of an honors course in differential geometry, and we welcome a discussion of additional ideas or suggestions.

Using visual blocks to support mathematical syntax

Anthony Morphett

Undergraduate mathematics students are expected to master the use of mathematical notation, including quantifiers and logical connectives. However, the language of mathematics and its syntax are rarely taught explicitly and formally in undergraduate courses; instead, students are expected to learn syntax informally and internalise syntactic rules through use, which can be problematic. Taking inspiration from projects such as Scratch (https://scratch.mit.edu/), one potential way to support students' use of syntax is with visual blocks. I will describe and demonstrate a system for building and manipulating mathematical expressions using virtual blocks. Graphical rules govern the ways in which blocks may be connected, and mirror the rules of mathematical syntax to ensure that only syntactically valid expressions may be constructed. Syntactical concepts such as type are represented visually using colour and shape. The system is based on the open-source Blockly project (https://developers.google.com/blockly/).

Tools for Teaching Computational Mathematics

Mr Matt Skerritt, Jon Borwein

"An Introduction to Modern Mathematical Computation" is a 2nd year subject intended to act as an introduction to computer assisted mathematics at the University of Newcastle, NSW, Australia. We discuss the design and execution of the course and two accompanying textbooks (one written for Maple code, and one for Mathematica code). We also discuss how the course has evolved over prolonged teaching.