From Scribal to Digital Schools, an Inspiring Journey in Mathematics (Education)

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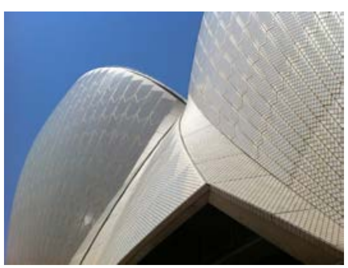


Seminar *Tools and Mathematics: Instruments for Learning*
29 Nov. – 1 Dec. 2016, CARMA, University of Newcastle, Sydney

2016, hope and tears…

Luc Trouche 4 janvier 2016 14:48
À : John David Monaghan, Jon Borwein
About the power of symmetry

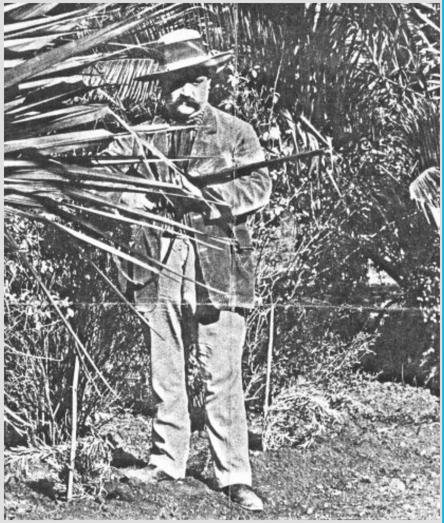
(only for French speaking geometers…)
With my best wishes for you, our book and your projects,
Amitiés,
Luc

Jon Borwein 5 janvier 2016 01:36
À : Luc Trouche, Judi Borwein, Naomi Borwein
Rép : About the power of symmetry 1

Excellent!

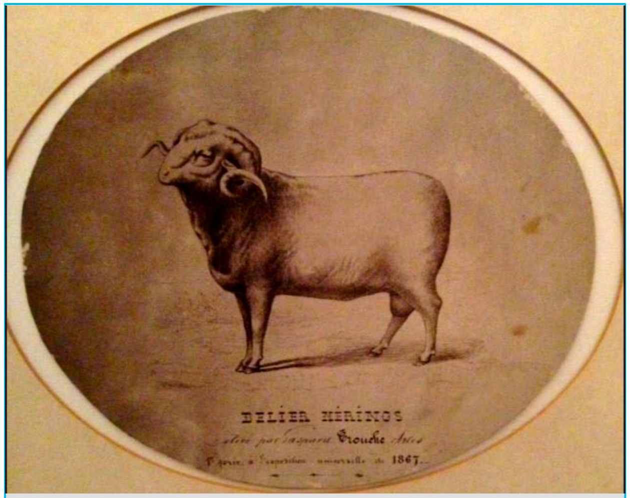
Cheers, Jon

JMB in cyberspace



Gaspard Trouche,
my great grandfather

A personal-familial link with Sydney. Actually a history of experimental approaches, search for resources, and professional development…



First prize at the Universal Exhibition of Paris in 1867

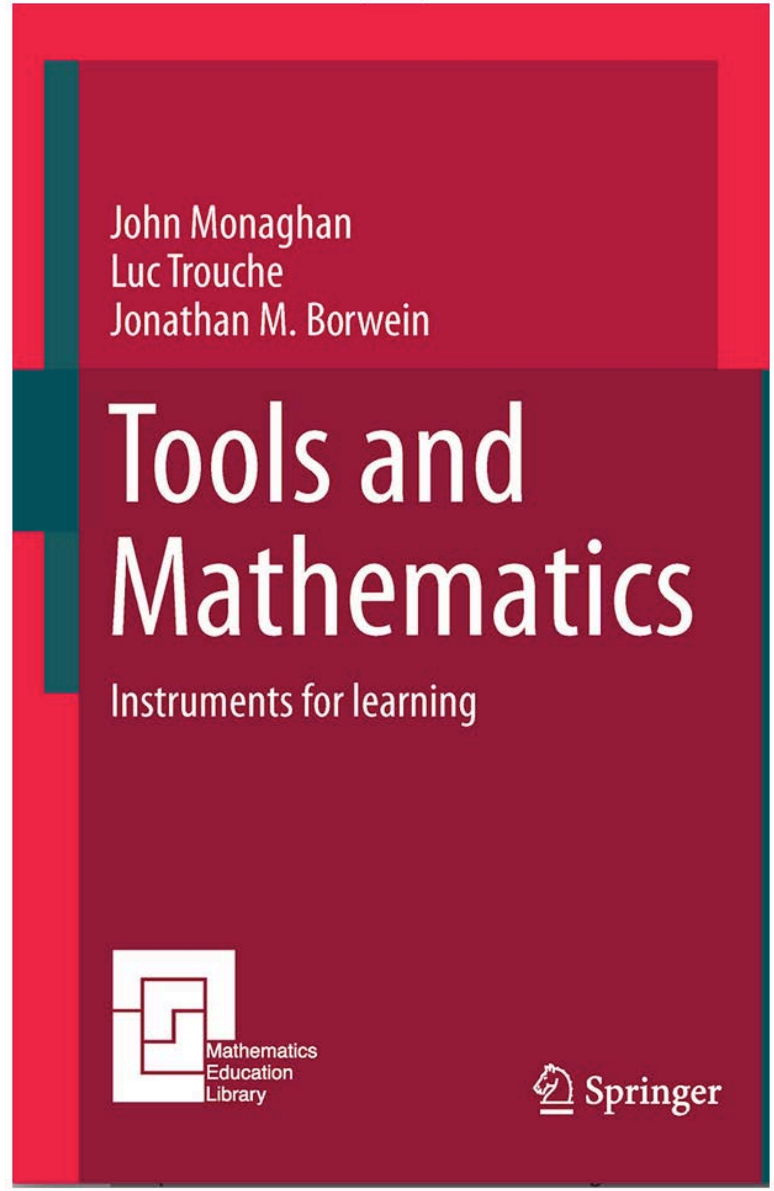
Purpose

Making a reflective meta analysis on tools in mathematics (education), from the book and my own experience, where issues of mathematics, didactics and tools are deeply interrelated.

Mathematics (education): learning *in* doing mathematics vs. learning *for* doing mathematics

Mathematics practices vs. school mathematics practices

Masters' resources vs. students' resources



Outlines

Trouche, L. (2016). *The Development of Mathematics Practices in the Mesopotamian Scribal Schools. Tablets and tokens, lists and tables, wedges and digits, a complex system of artefacts for doing and learning mathematics, 2000 years BCE* [collaboration with C. Proust]

Instrumentation vs. instrumentalisation

On the scribal schools side

- A joint emergence of the art of writing, the art of computing and schools
- Tools shaping writing, computing and schools [and vice-versa]
- Looking at scribal schools resources, usages, and computations

On the "digital" schools side

- Proofs without words / with a lot of resources
- Teaching as co-designing

On the scribal schools side

A joint emergence of the art of writing, the art of computing and schools

8500 BCE | 3500 BCE | 3000 BCE | 2000 BCE

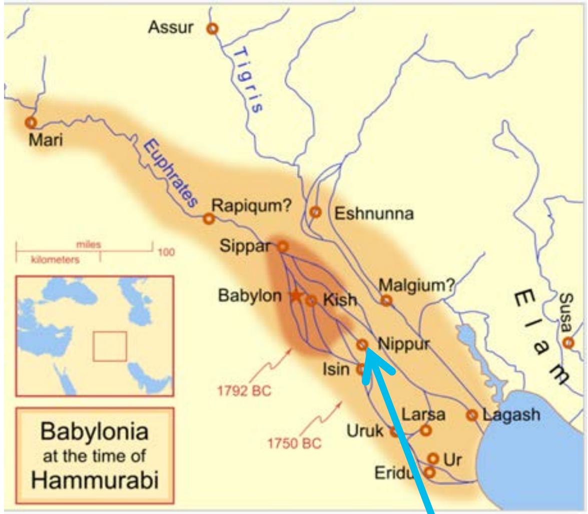
From 12 to 350 shapes of token representing “things” to be counted (sheep, jar of oil, garment…)

“Abstract” tokens (standing for 1 or 10) and traces on clay envelops (for trading purpose)

Written numbers on clay tablets, abstract numbers and concrete measures (Schmandt-Besserat 2009)

On the scribal schools side

A joint emergence of the art of writing, the art of computing and schools



The institutions where these computations took place, the *scribal schools*, co-emerging with writing and mathematics (as a system of signs and techniques)

A period and a place exceptionally favourable for historians, due to the huge quantity of school tablets handed down to us. No other educational system of the distant past is as well documented as that of Mesopotamia

This situation is due to the material used for building the tablets: the clay, a nearly indestructible material

It also ensues from the reuse of dry and waste tablets as construction material. Trapped in walls, floors or foundations of houses, tablets produced by masters and students and subsequently discarded have survived till today…

On the scribal schools side

A joint emergence of the art of writing, the art of computing and schools

A poem celebrating writing, knowledge and tools (for writing / for measuring)

"Nisaba, the woman radiant with joy,
The true woman scribe, the lady of all knowledge,
Guided your fingers on the clay,
Embellished the writing on the tablets,
Made the hand resplendent with a golden calame
The measuring rod, the gleaming surveyor's line,
The cubit ruler which gives wisdom,
Nisaba lavishly bestowed upon you"

Nisaba is the goddess of schools, scribes and mathematicians (-2000, -1500)

Proust, 2007, p. 55

On the scribal schools side

Tools shaping writing, computing and schools

The Sumerian name for ‘tablet’ is DUB, for ‘scribe’ is DUB.SAR, meaning *the one who writes on tablets,* for ‘ scribal school’ is É.DUB.BA, meaning *the house of the tablets* (dictionnary)

- Tablets, made of clay already used for other purposes (cooking, arts…), fresh clay being an efficient support for writing
- *Calame* (meaning ‘pen’ in Arabic), i.e. a piece of reed, sometimes of bone or ivory, or wood

Nippur, 2000 BCE

Wikipedia…

On the scribal schools side

Tools shaping writing, computing and schools

The calame: specially rounded at first and bevelled thereafter. The incision of this artefact in fresh clay makes it difficult to draw curves and encourages the user to draw triangles and short segments

The incision of signs on a malleable media gives not a flat writing like that obtained with ink and paper, but an embossed writing

Signs should be read with lighting that allows the reader to identify all incisions

Nippur, 2000 BCE

On the scribal schools side

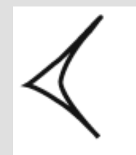
Tools shaping writing, computing and schools

‘Economical’ reasons lead to write digits with a minimum number of signs, actually 2

These two signs recall those used 1000 years ago for designing 1 (a vertical wedge) and 10 (circular)

These similarities evidence, for writing numbers, a deep continuity over the time in this region (due to the communications between people for trading purpose… and scientific exchanges)

A vertical wedge standing for 1



An oblique wedge standing for 10

Iran, 3100 BCE

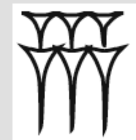
On the scribal schools side

Tools shaping writing, computing and schools

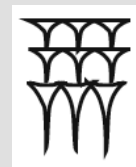
These signs were aggregated by a maximum of three figures, for allowing a rapid reading

Then an additive principle allows to write ‘intermediate’ numbers

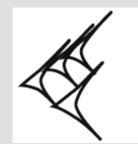
The beginning of a an efficient *system of artefacts*: these artefacts, both material (clay tablets and calame) and symbolic (figures and rules of displaying them), allow to write digits from 1 to…



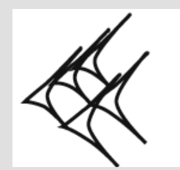
Standing for 6



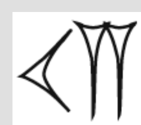
Standing for 9



Standing for 40



Standing for 50



Standing for 12

On the scribal schools side

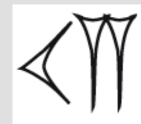
Tools shaping writing, computing and schools

… 59, due to the invention (for economical reasons again) of a positional numeration system, mostly a sexagesimal one

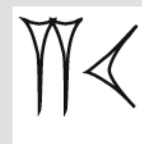
In mathematical texts, the numbers are made of sequences of digits following a *positional principle* in base 60

Each sign noted in a given place represents 60 times the same sign noted in the previous place (on its right)

The concatenation has to be considered carefully…

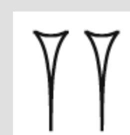


stands for 12, but…



…stands for
2 x 60 + 10 = 130
We will note it as 2.10

stands for 2, but…



…stands for
1 x 60 + 10 = 70
We will note it as 1.10

On the scribal schools side

Tools shaping writing, computing and schools

In the Old Babylonian period, the cuneiform writing did not allow to distinguish 1 and 1.0. In most of the situations, the context allows to interpret: it was a *floating notation*

This ambiguity of the notation could created errors, for example for distinguishing 12, or 10.2

It was corrected in later period by the use of a new sign  to denote the different levels.

stands for 12, but…

…stands for 10.2 = 602

On the scribal schools side

Tools shaping writing, computing and schools

Now we are able to understand the content of this clay tablet…

- 7.35
- 7.35
- 57.30.25

Leading to three questions:

- Is it a list of numbers, or a computation?
- In case of a computation, is it true?
- And how this kind of computation was performed?

Nippur, 2000 BCE

On the scribal schools side

Tools shaping writing, computing and schools

Second question: is it true that

7.35 x 7.35 = 57.30.25 ?

As the ‘raison d’être’ of my own today orchestration is: facilitating your appropriation of the Mesopotomian computation spirit…

… I propose to introduce a new artefact, a digital one, for checking this multiplication, and having a walk into Mesopotamian mathematics digits.

MesoCalc

A Mesopotamian Calculator

http://baptiste.meles.free.fr/site/mesocalc.html (lien)

B. Mélès (CNRS, Université de Lorraine), et C. Proust (CNRS, Université Paris-Diderot)

On the scribal schools side

Tools shaping writing, computing and schools

Third question: how this kind of computation was performed?

No traces of intermediate computation on “draft tablets”…

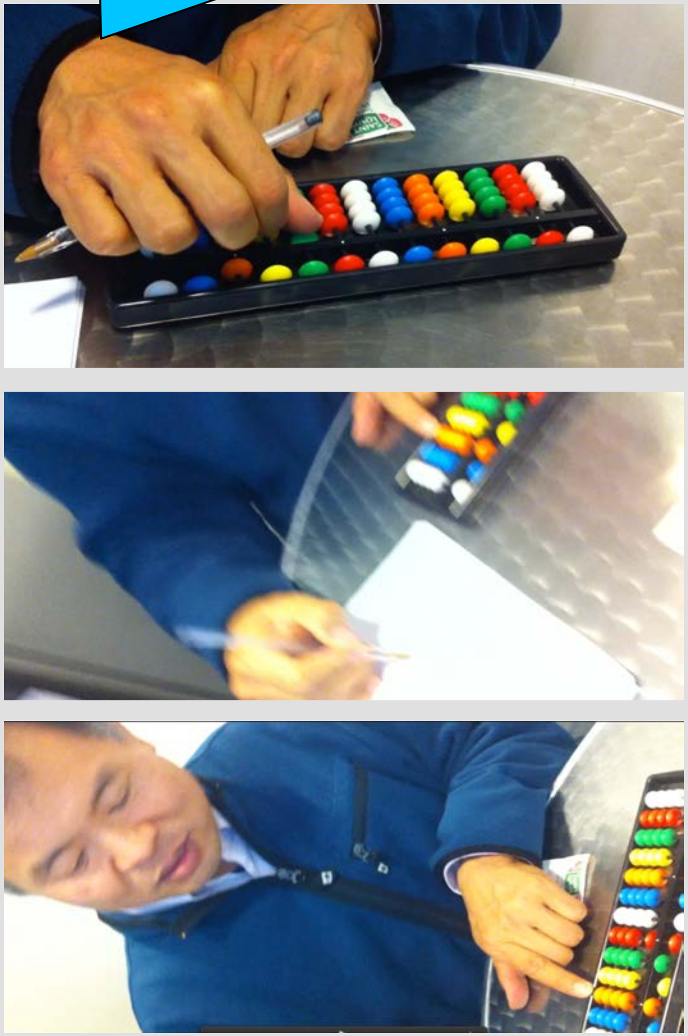
No written algorithm described

Strong hypothesis of an artefact dedicated to such computation out of the tablet, an abacus with token:

- From texts written on tablets
- From excavations evidencing the simultaneous presence of tablets and token

Computation as a flexible combination of artefacts

A Japanese colleague using his right hand for keeping a pen (and writing results on a sheet) and moving balls on an abacus



On the scribal schools side

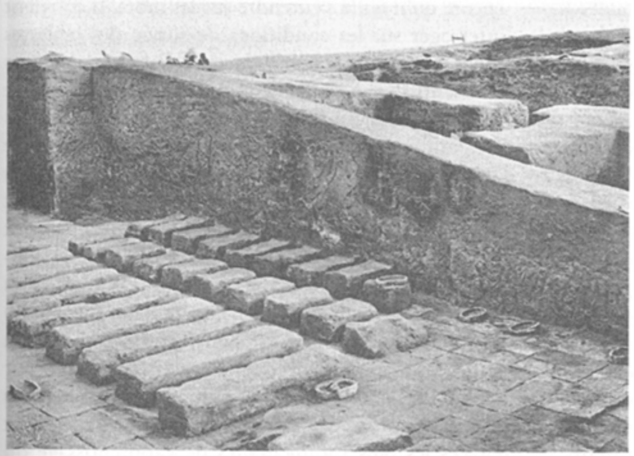
Tools shaping writing, computing and schools

Tools as a total social fact, incorporating a complex anthropological reality (John lecture)

What the scribal schools looked like? The first “natural” hypothesis was that… they looked like our schools (Charpin, 2008)

But, remember, the cuneiform signs should be read with lighting that allows the reader to identify all incisions in order to avoid misinterpretation.

Then one can hypothesis that these schools took place in open air, hypothesis confirmed now by the scribal school literature itself



Archaeological excavations in a paleo-mesopotamian site: a school?

Today, a school outside the walls, in Timbuktu

On the scribal schools side

Looking at scribal schools resources, usages, and computations

A complex system of clay tablets of different natures (metrological lists and tables, reciprocal tables, square roots tables…)

Writing tools and measuring tools

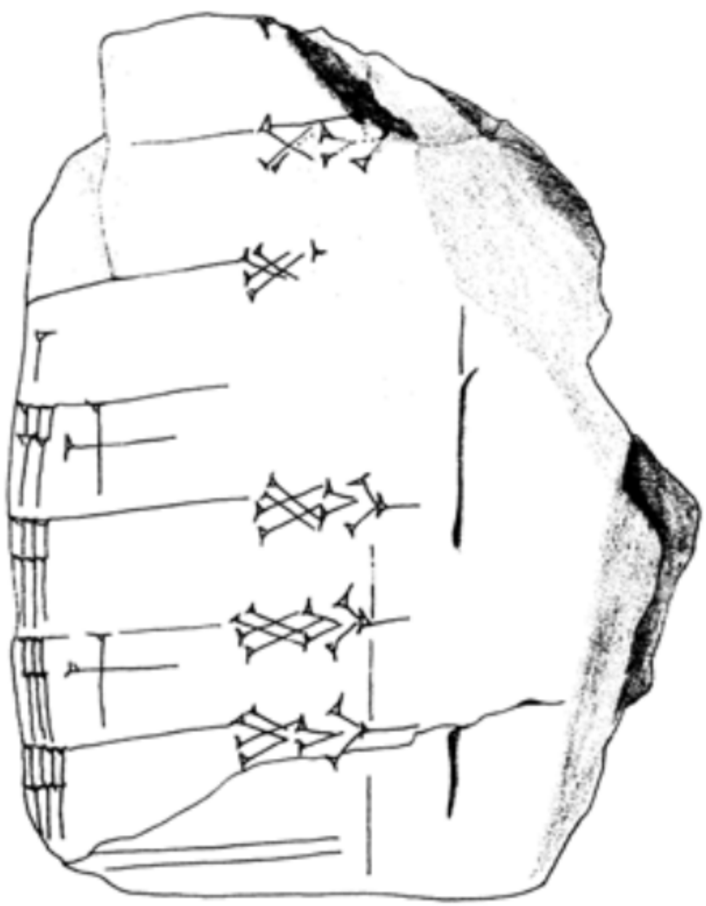
Metrological lists	Capacity list
	Weight list
	Surface list
	Length list
Metrological tables	Capacity table
	Weight table
	Surface table
	Length table
	Height table
Division/multiplication tables	Reciprocal table
	Multiplication tables
	Square table
Tables of roots	Square root table
	Cubic root table

On the scribal schools side

Looking at scribal schools resources, usages, and computations

“In a first step, the students learnt to write short excerpts, reproducing a model on the obverse of tablets, then they memorised the pronunciation, they recited the excerpt, and, in the last step, they reproduced by heart a large part of the list by writing it on the reverse of a tablet.

Learning therefore inextricably combined writing and memorisation” (Proust, 2012)



On the scribal schools side

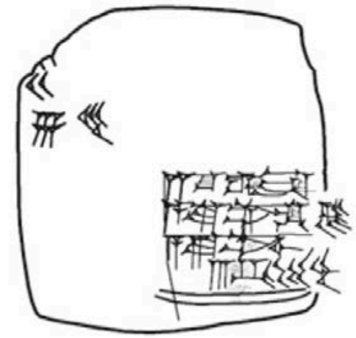
Looking at scribal schools resources, usages, and computations

A complex interaction between ‘abstract’ numbers and concrete measures…

Jana and the fractions:

2/3 x *1/2*

Tablet UM 29-15-192 (Neugebauer & Sachs 1984)



Hand copy made by Proust, personal communication

UM 29-15-192 -Transcription	**Translation**	**Interpretation**
[2]0 20 6.40	20 x 20 = 6.40	
2 šu-si ib$_2$-si$_8$ --------------------- a-ša$_3$-bi en-nam --------------------- a-ša$_3$-bi igi-3-gal$_2$ še-kam ==============	2 šu-si the side of the square What is its area? Its surface is 1/3 še [a *šu-si* (= a finger) is a length measuring unit a *še* (= a grain) is an area measuring unit]	2 šu-si → 20 20 x 20 = 6.40 6.40 → 1/3 še

On the scribal schools side

Looking at scribal schools resources, usages, and computations

Computation of the reciprocal of A = 25.18.45 (tablet CBS 1215)

A multi-column tablet containing advanced mathematics evidences the fact that the masters were not only teaching elementary mathematics.

They worked also as scholars, for developing mathematics, exchanging texts between masters across the different schools, insuring the development of a common body of knowledge and artefacts in scribal schools, over a large territory

On the scribal schools side

Looking at scribal schools resources, usages, and computations

The computation of a reciprocal only concerns regular numbers, i.e. in the sexagesimal numeration, numbers that are products of powers of 2, 3, and 5: that is the case for A = 25.18.45

The goal of the algorithm is to decompose the regular number at stake as the product (non unique) of regular numbers whose reciprocal is well known (this algorithm lies therefore on the property: ‘ the reciprocal of a product of numbers is the product of the reciprocals of these numbers’)

The ‘ well known reciprocals’ come from a table, part of the curriculum.

2	**3**	**4**	**5**	**6**	**8**	**9**	**…**	**16**	**…**	**45**	**…**
30	20	15	12	10	7.30	6.40	…	3.45	…	1.20	…

On the scribal schools side

Looking at scribal schools resources, usages, and computations

The second property supporting the algorithm is: ‘if a regular number terminates the writing of A , then it is a regular factor in one decomposition’. Now we can begin the computation:

- First step, we isolate, in the final digits of A (thinking A as 25.15 + 3.45), a number present in the table (3.45), which reciprocal is 16
- Second step, we try to write A as a product of n and 3.45; the number n is therefore equal to A x16 (which is the reciprocal of 3.35), i.e. n = 6.45
- Then A = 6.45 x 3.45

2	3	4	5	6	8	9	…	16	…	45	…
30	20	15	12	10	7.30	6.40	…	3.45	…	1.20	…

On the scribal schools side

Looking at scribal schools resources, usages, and computations

A = 6.45 x 3.45… Then we apply the same algorithm for 6.45

- First step, we isolate, in the final digits of this number, a number present in the table: 45, which reciprocal is 1.20.
- Second step, we try to write 6.45 as a product of m and 45; the number m is therefore equal to 6.45 x 1.20 (which is the reciprocal of 45), i.e. m = 9.

The number 9 is present in the table of reciprocals, here is therefore the end of the algorithm : A = 3.45 x 45 x 9 then 1/A = 16 x 1.20 x 6.40 = 2.22.13.20

2	3	4	5	6	8	9	…	16	…	45	…
30	20	15	12	10	7.30	6.40	…	3.45	…	1.20	…

A = 3.45 x 45 x 9 then 1/A = 16 x 1.20 x 6.40 = 16 x 8.53.20 = 2.22.13.20

10.6.48.5**3.20** 18
3.2.2.**40** 22.[30]
1.8.**16** 3.4[5]
4.**16** 3.[45]
16 3.[45]
1[4.3.4]5
52.44.[3.4]5
19.46.31.24.22.[30]
5.55.57.25.18.**4[5]** 16
1.34.55.18.**45*** 16
25.18.**45*** [16]
6.45 [1.20]
9 [6.40]
8.53.20
2.22.13. 20
37.55.33.20
10.6.48.53.20

On the scribal schools side

A preliminary conclusion

A continuous development of artefacts material (clay tablets, calame, tokens, table…) and symbolic (algorithms)

A combination of memorization, writing and manipulating

A flexible view on numbers (the floating notation and the switch between numbers and measures)

The role of masters for *orchestrating* (Trouche & Drijvers, 2014) students computation and developing mathematics

The importance of schools as a laboratory for developing tools, social computation practices and mathematics

Interlude

Clay tablets and tokens, old and new resources

The cohabitation between computation with Indian digits, and computation with abacus, during several centuries in France

The cohabitation between slide rule and calculator, two faces of a same artefact, during several years in France (1975)

Interlude

From scribal schools to ‘digital’ schools

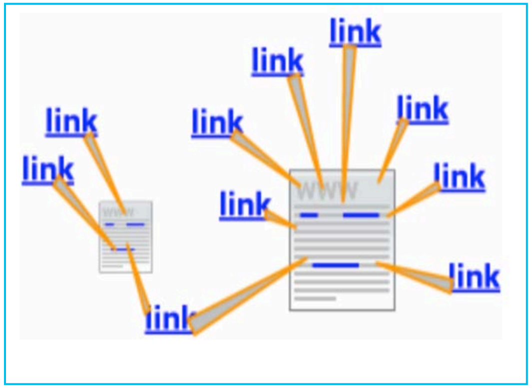
A huge difference about “who are the learners”, From aristocratic education to education for all

2000 BCE vs. 2000 ACE (today) two critical moments for information, communication and knowledge

In both cases, an upheaval of the support of knowledge (from oral to written supports vs. from paper to digital supports)

A change of dimensions for thinking: oral (1D); written (2D); digital (3D)

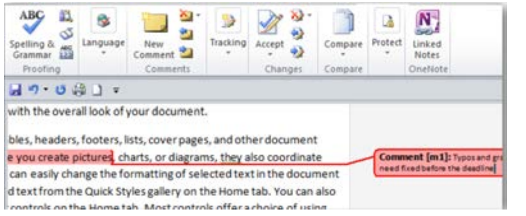
(Bachimont, 2010)

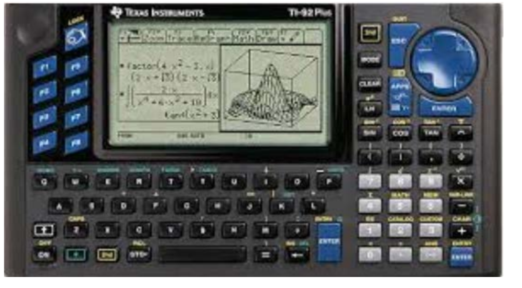


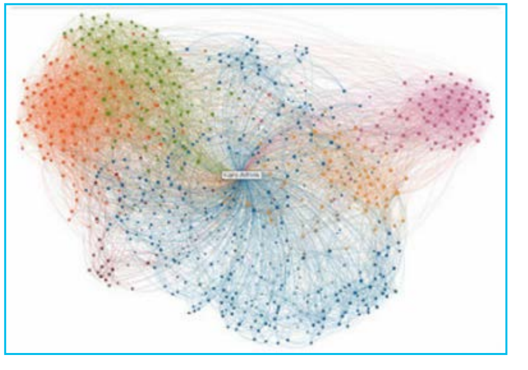
Interlude

Digital tools…

- … shaping writing: new interactions reading-writing, using-designing
- … shaping computing: cf. “The calculator debate” (John in the book)
- … shaping things (Michael presentation);
- … shaping schools: just the beginning of a process, cf. the MOOC’s story
- More resources, more complex orchestrations, examples in the book…







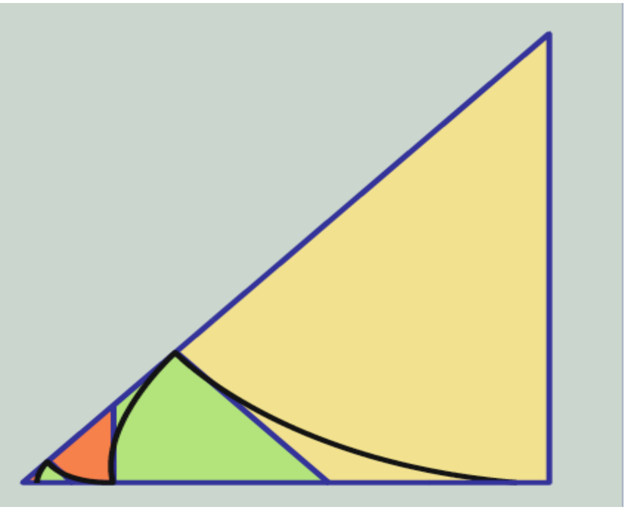
Proofs without words / with a lot of resources?

…"Classic proofs without words, though most such proofs benefit from a few words of commentary" (Borwein, 2016, p. 31)

One example : √2 is irrational, as assumed by Tom Apostal (1923-2016), and the proof is here

Can you prove it… only with a few words?

Or in using a variety of artefacts ?



Assume the large triangle is the smallest 45◦ right-angled triangle with integer sides. The complement of the brown kite is a smaller such triangle

Proofs without words / with a lot of resources?

$$\frac{a}{b} = \sqrt{2}$$

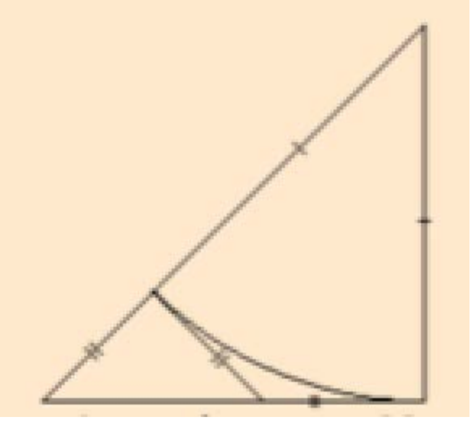
a and be integers, choosing the smallest value for a and b (idea of uniqueness)

$$\frac{a^2}{b^2} = 2$$

Moving to rational numbers

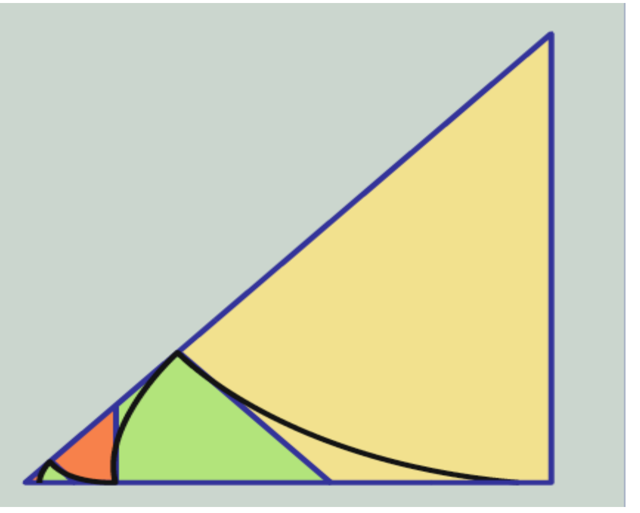
$$a^2 = 2 \times b^2$$

Moving to integers



From a numerical to a geometrical frame (or point of view)

Paper/pencil, or DGS; + coding

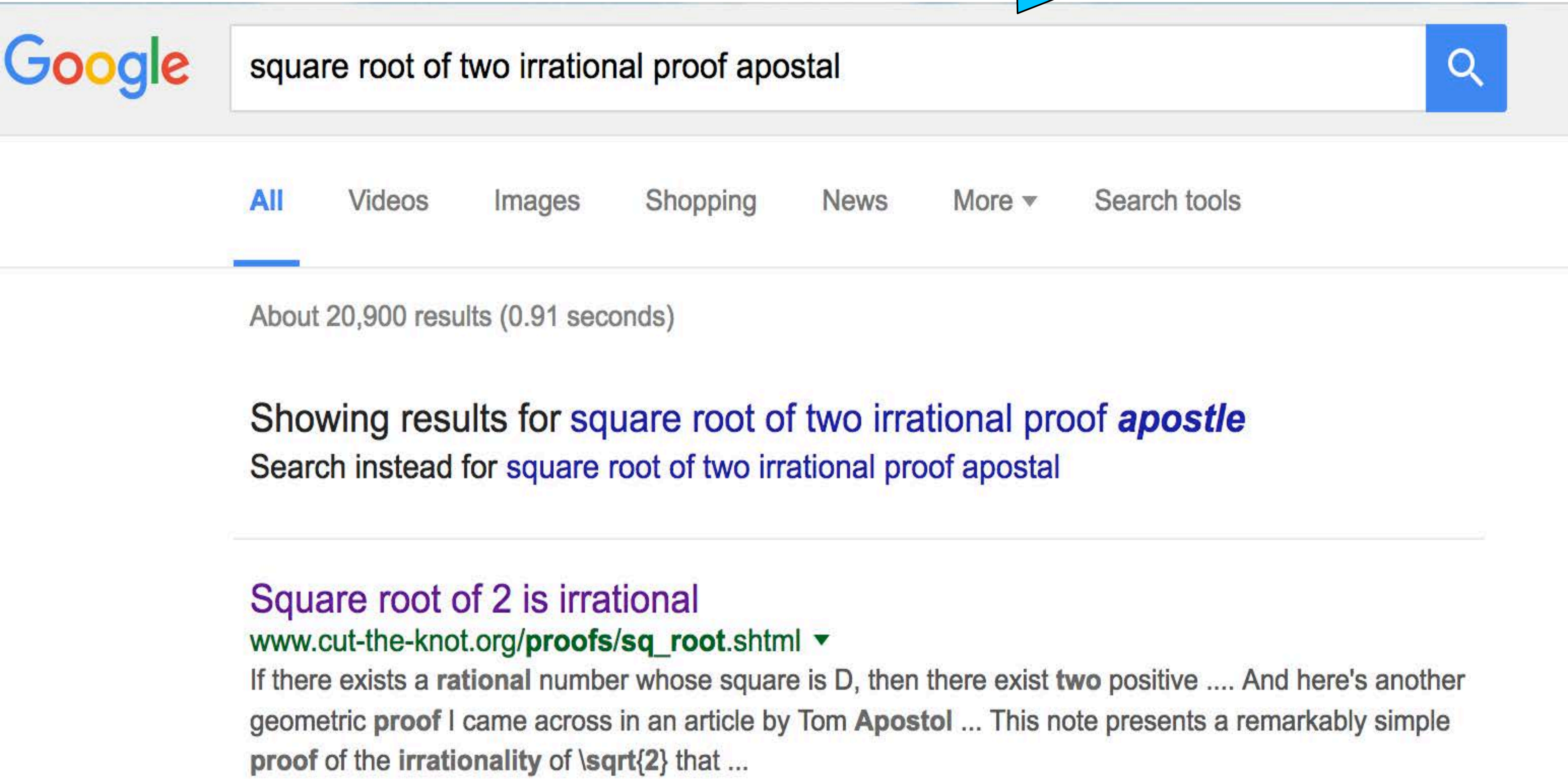


Assume the large triangle is the smallest 45◦ right-angled triangle with integer sides. The complement of the brown kite is a smaller such triangle

On the digital school side

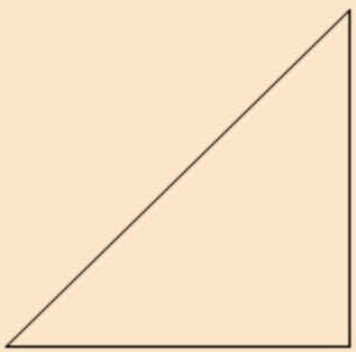
Proofs without words / with a lot of resources?

Possible to have (extra) information via Internet with a request more or less accurate…

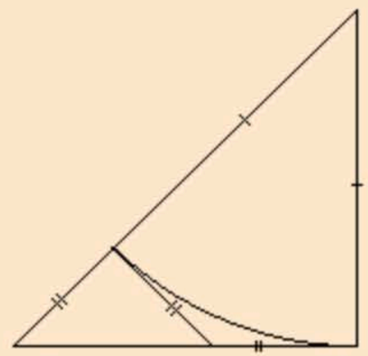


This note presents a remarkably simple proof of the irrationality of $\sqrt{2}$ that is a variation of the classical Greek geometric proof.

By the Pythagorean theorem, an isosceles right triangle of edge-length 1 has hypotenuse of length $\sqrt{2}$. If $\sqrt{2}$ is rational, some positive integer multiple of this triangle must have three sides with integer lengths, and hence there must be a smallest isosceles right triangle with this property. But inside any isosceles right triangle whose three sides have integer lengths we can always construct a smaller one with the same property, as shown below. Therefore $\sqrt{2}$ cannot be rational.



If this is an isosceles triangle with integer sides



then there is a smaller one with the same property

***Construction**. A circular arc with center at the uppermost vertex and radius equal to the vertical leg of the triangle intersects the hypotenuse at a point, from which a perpendicular to the hypotenuse is drawn to the horizontal leg. Each line segment in the diagram has integer length, and the three segments with double tick marks have equal lengths. (Two of them are tangents to the circle from the same point.) Therefore the smaller isosceles right triangle with hypotenuse on the horizontal base also has integer sides.*

The reader can verify that similar arguments establish the irrationality of $\sqrt{n^2+1}$ and $\sqrt{n^2-1}$ for any integer $n > 1$. For $\sqrt{n^2+1}$ use a right triangle with legs of lengths 1 and n. For $\sqrt{n^2-1}$ use a right triangle with hypotenuse n and one leg of length 1.

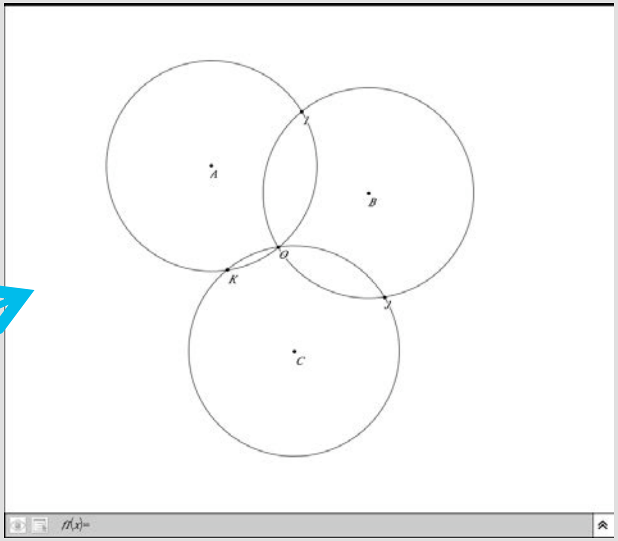
Proofs without words / with a lot of resources?

Three circles have the same radius R, and pass through the same point O.

What about the three other intersections points I, J and K?

Can you prove it… only with a few words?

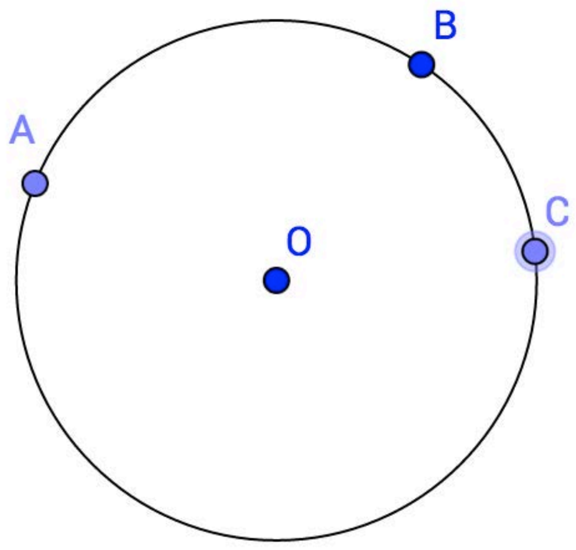
Or in using a variety of artefacts ?

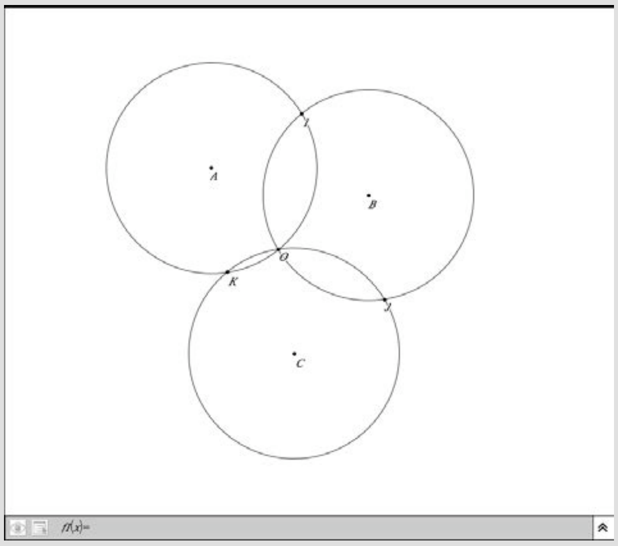


I, J and K are on a circle whose radius is R, and the proof is in the figure itself

Proofs without words / with a lot of resources?

First step, constructing a figure with a DGS, easier (?) than with paper-pencil





I, J and K are on a circle whose radius is R, and the proof is in the figure itself

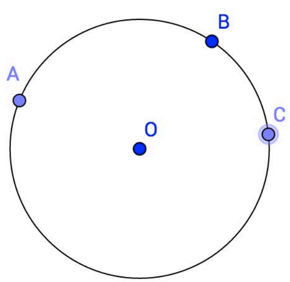
On the digital school side

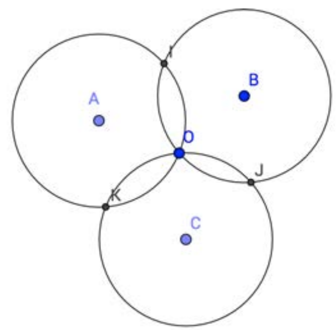
Proofs without words

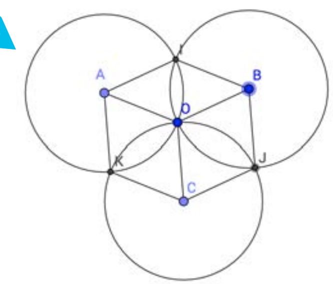
Strategies / tactics (Tristam): get hands dirty, make it easier, recognize patterns

Constructing the figure, needing to have a synthetic view of the configuration

Evidencing, in the figure itself the geometrical property, moving the free points… and moving from a 2D view to a three D view

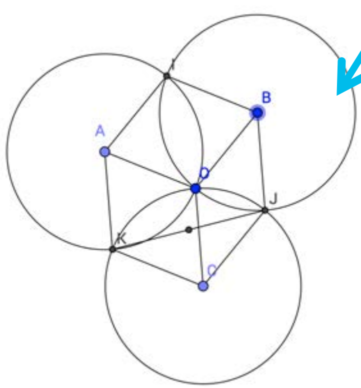


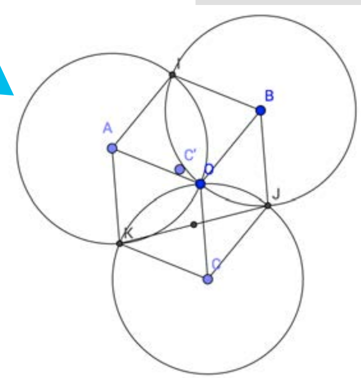


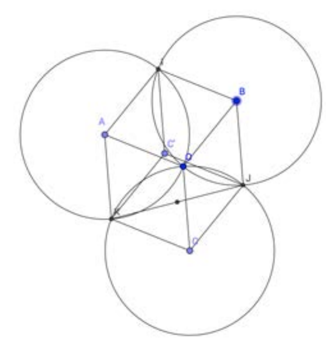


Constructing the missing point

Finishing the “cube”, checking, summarizing the whole process, communicating…







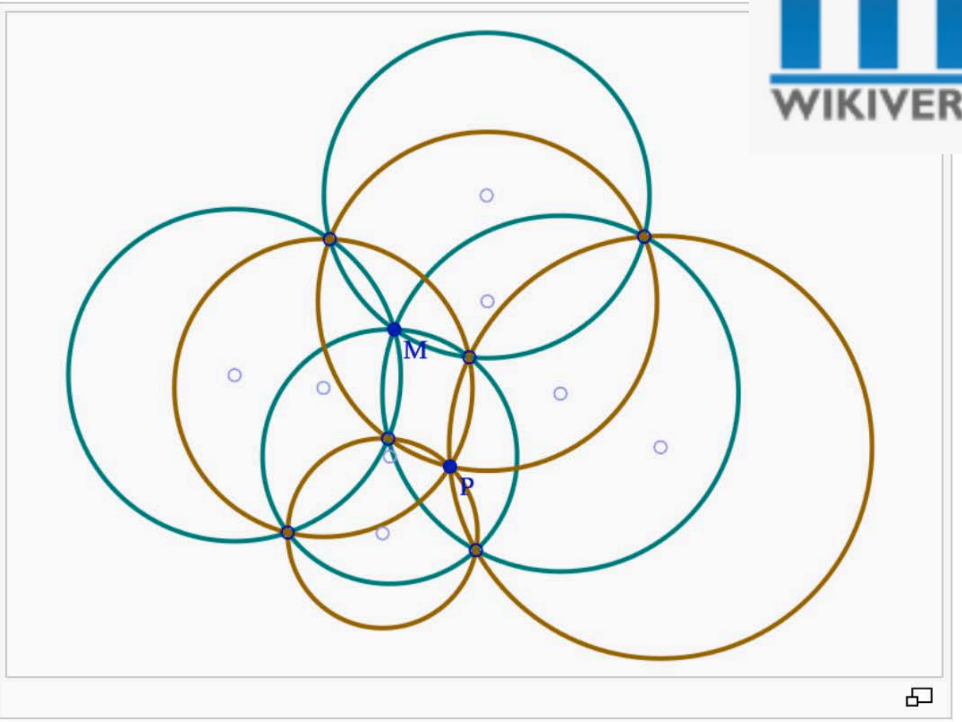
Proofs without words / with a lot of resources?

https://en.wikiversity.org/wiki/Clifford's_circle_theorems

Clifford's circle theorems

In geometry, **Clifford's theorems**, named after the English geometer William Kingdon Clifford, are a sequence of theorems relating to intersections of circles.

The first theorem considers any four circles passing through a common point *M*. Four new circles can be constructed to pass through their other intersection points taken in triplets, so that each intersection's triple touches three circles only. Then these four circles also pass through a single point *P*.



Proofs without words / with a lot of resources?

> “Sometimes, it is easier to see than to say” (Naomi Borwein)

> « Magie et image ont même lettres et ce n’est que justice » (Debray, 1992)

For a given learning objective, thinking mathematical problems, tools and orchestration for moving from seeing to saying… and vice-versa

Balancing seeing, saying, making, memorizing, thinking…

The difficult way towards the “raisons d’être”.

On the digital school side

Teaching as co-designing

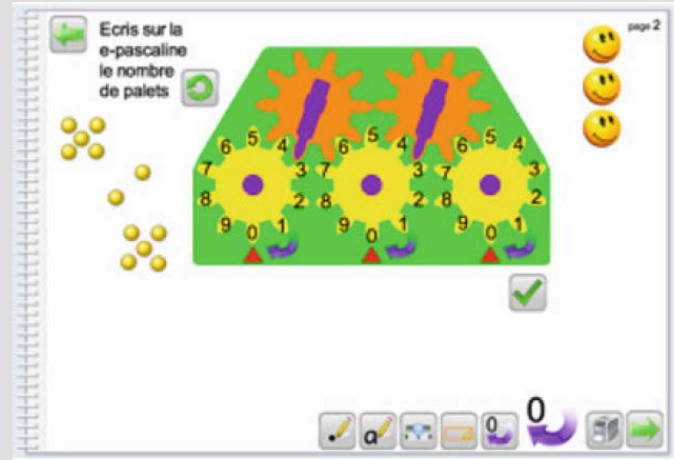
See the Panel on "thinking, moving and feeling mathematically"

Need for specific mathematics situations taking profit of material and digital resources (= an open resource system, due to Internet)

Need for orchestrations combining various resources

Preparation, incubation, illumination, and verification (Hadamard, 1945)

Teachers as designers, needing specific knowledge and time, then the necessary collaboration of masters



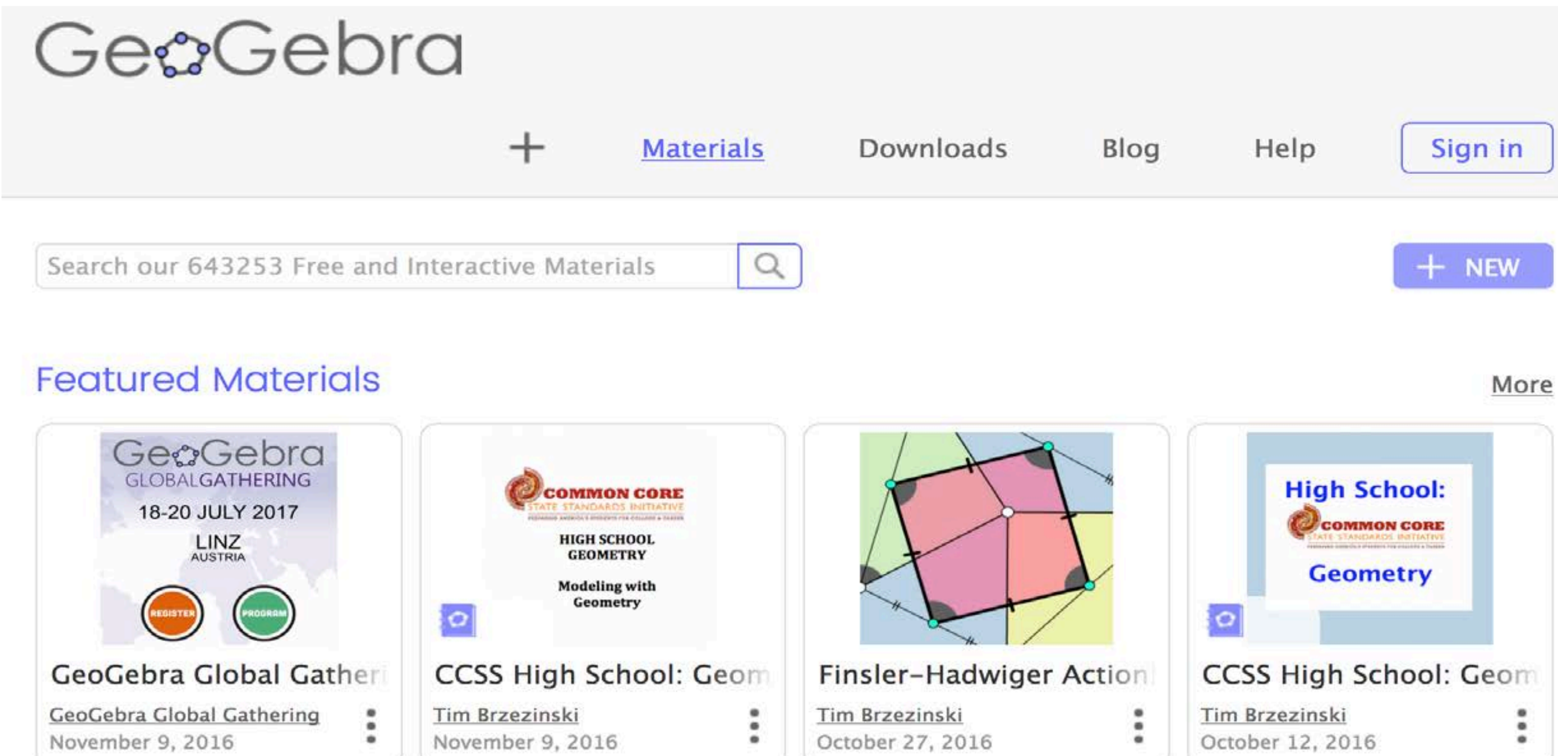
A combination of two twin artefacts, digital tablet vs. tangible device (Maschietto & Soury-Lavergne 2013)

On the digital school side

Teaching as co-designing

Australian GeoGebra Institute
Aims: Training and Support;
Development and Sharing;
Research and Collaboration

Geogebra communities: math teachers sharing a huge number of resources and contributing to the development of the software itself



On the digital school side

Teaching as co-designing

Looks like the experience of the French IREM (Institutes of research on mathematics teaching

The Australian case:

AMSI: “AMSI Schools implements *community projects for clusters of schools* supported by AMSI Specialists through a program of professional development sessions and school visits”

MESIG: “creatively bring together practitioners and researchers with common interests in education in the mathematical sciences, and in particular how our tool use influences our thinking”.

Teaching as co-designing

The French case: Sesamath (5000 teachers, 100 working groups, designing e-textbooks and exercises used by 100000 teachers…). A platform including a laboratory for steering teachers collaboration

Announcing a new period for free e-textbooks? Teachers as designers of their own resources? Teachers Life Long Learners?

Mathematics for everybody

Working together, supporting one another, communicating

On the digital school side

For me, resulting from this reflective activity

The need for better knowing, and taking into account, all the artefacts of the school ecosystem (old and news, students' ones and teachers' ones…)

The need for better balancing memorization, routines, manipulating and investigating (subtle orchestrations to be thought)

The need for considering teachers as co-designers of their own resources

The need for considering schools as the place where teaching and learning happen (in and out of the wall of the 'official schools'…)

What is the heart of teacher's work? « Authentic teaching » (Sitti) or working on resources?

Today, nothing is really new, and all is actually new…

From clay tablets to digital ones

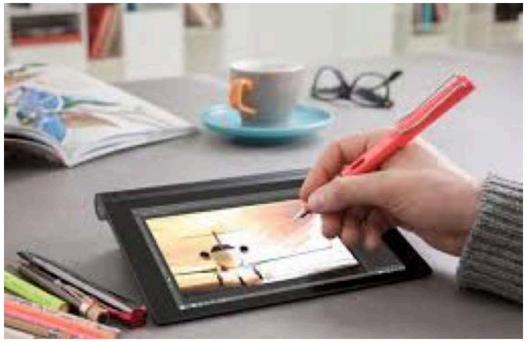
Some elements of the ancient time…

Back to the calame

Back to the use of the finger

Back to the oral communication

What traces of the today digital tablets will be found… in 4000 years?



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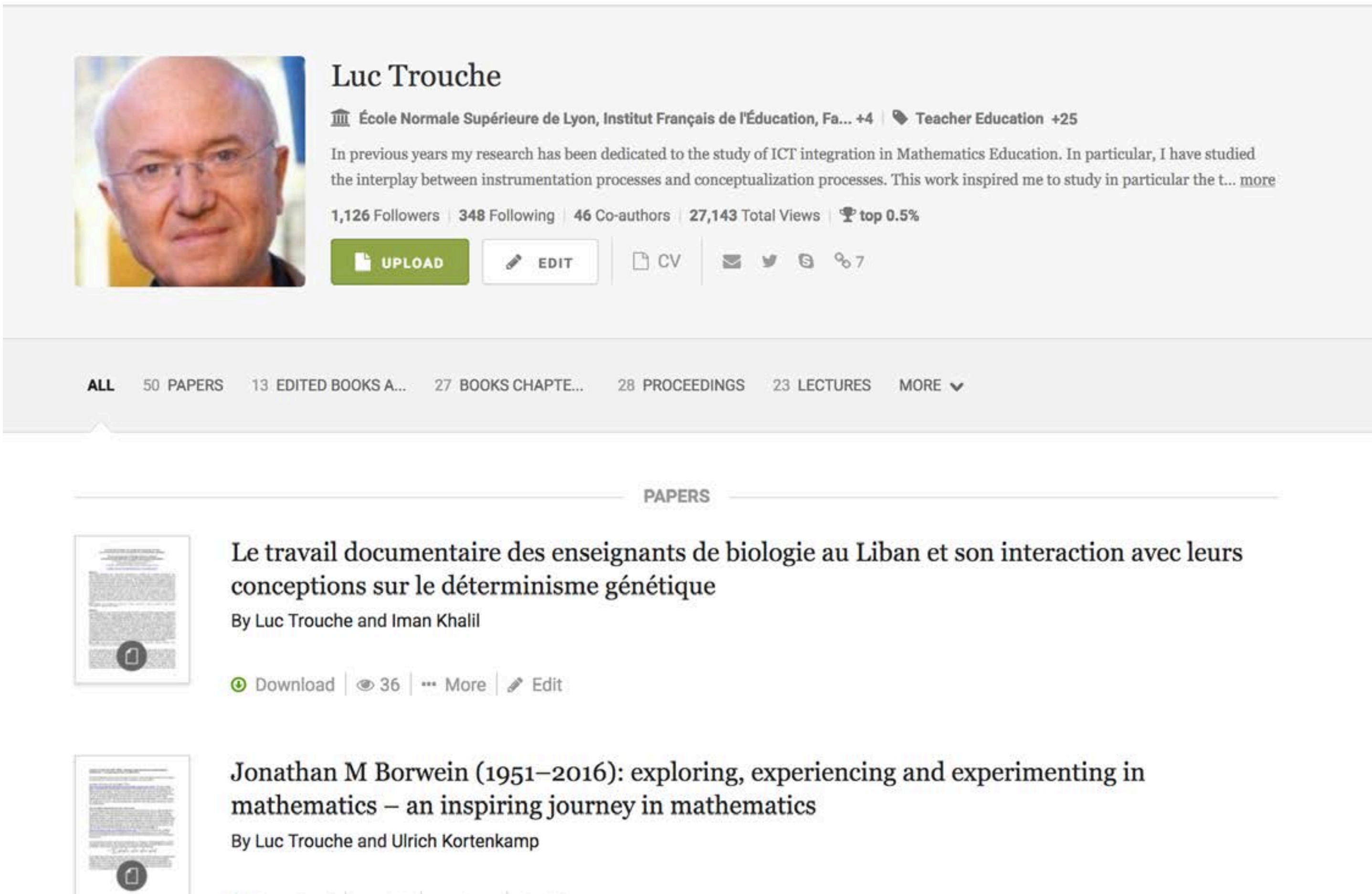

Remembrance Day

Dedicated to the memory of
Professor Jonathan Michael Borwein

Friday 10 February 2017, Paris,
Institut Henri Poincaré

- David H. Bailey (Berkeley National Laboratory)
- Patrick Combettes (North Carolina State University)
- Ivar Ekeland (University Paris-Dauphine)
- Martin Grötschel (Berlin-Brandenburgische Akademie der Wissenschaften)
- Adrian S. Lewis (Cornell University)
- Luc Trouche (Ecole Normale Supérieure, Lyon)
- Qiji Jim Zhu (Western Michigan University)
- Matthew K. Tam (University of Göttingen and CARMA)
- F.J. Aragón Artacho (University of Alicante and CARMA)
- Alexander Ioffe (Technion)

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