



Experimental Mathematics in Action

Rough Schedule

- **Five morning lectures** (2 hrs with break in middle). They will more-or-less correspond as follows:
 - L1-L2: Chapter 1
 - L2-L3: Chapters 2 and 3
 - L4: Chapters 5 and 7
 - L5: Chapter 8
- **Four hands on afternoons.** (Maple, Mma or web-based)
- All **resources** are at <http://ddrive.cs.dal.ca/~isc/portal>



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“Experimental mathematics has not only come of age but is quickly maturing, as this book shows so clearly. The authors display a vast range of mathematical understanding and connection while at the same time delineating various ways in which experimental mathematics is and can be undertaken, with startling effect.”

—Prof. John Mason, Open University and University of Oxford

Experimental Mathematics in Action

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The last twenty years have been witness to a fundamental shift in the way mathematics is practiced. With the continued advance of computing power and accessibility, the view that “real mathematicians don’t compute” no longer has any traction for a newer generation of mathematicians that can really take advantage of computer-aided research, especially given the scope and availability of modern computational packages such as Maple, Mathematica, and MATLAB. The authors provide a coherent variety of accessible examples of modern mathematics subjects in which intelligent computing plays a significant role.

“Computing is to mathematics as telescope is to astronomy: it might not explain things, but it certainly shows ‘what’s out there.’ The authors are expert in the discovery of new mathematical ‘planets,’ and this book is a beautifully written exposé of their values, their methods, their subject, and their enthusiasm about it. A must read.”

—Prof. Herbert S. Wilf, author of *generatingfunctionology*

“From within the ideological blizzard of the young field of Experimental Mathematics comes this tremendous, clarifying book. The authors—all experts—convey this complex new subject in the best way possible; namely, by fine example. Let me put it this way: Discovering this book is akin to finding an emerald in a snowdrift.”

—Richard E. Crandall, Apple Distinguished Scientist, Apple, Inc.

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Things Computer are Good For

- **High Precision Arithmetic: the microscope**
- **Formal Power-Series manipulations: $\theta_2 \theta_3^2$**
- **Continued Fractions: changing representations**
- **Partial Fractions : changing representations**
- **Pade' Approximations: changing representations**
- **Recursion Solving: 'rsolve' and 'gfun'**
- **Integer Relation Algorithms: 'identify'**
- **Creative Telescoping: Wilf-Zeilberger**
- **Pictures, Pictures, Pictures**

Ten Things to Try Them On, I

1. Identify

1.4331274267223117583171834557759918204315127679060

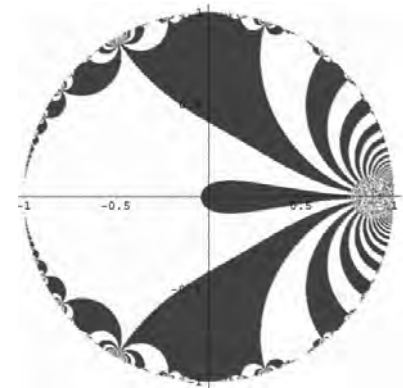
2. **Compute** the following to 50 digits for $N=1,2,3,4,5$ and **explain** the answer

$$4 \sum_{n=0}^{5 \cdot 10^N} \frac{(-1)^n}{2n+1}$$

3. **Find** the first three numbers expressible as the sum of two cubes in exactly two ways. The first is **$1729=12^3+1=10^3+9^3$** .

4. Evaluate

$$\int_0^1 \frac{x^4(1-x)^4}{1+x^2} dx$$



5. **Evaluate** for $\text{sinc}(x) = \sin(x)/x$

$$\begin{aligned} \frac{1}{2} &+ \sum_{n=1}^{\infty} \text{sinc}(n) \text{sinc}(n/3) \text{sinc}(n/5) \cdots \text{sinc}(n/23) \text{sinc}(n/29) \\ &= \int_0^{\infty} \text{sinc}(x) \text{sinc}(x/3) \text{sinc}(x/5) \cdots \text{sinc}(x/23) \text{sinc}(x/29) dx \end{aligned}$$

Ten Things to Try Them On, II

6. Evaluate

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1} k^2}{k^3 + 1}$$

7. and 8. Determine

$$\sum_{n=1}^{\infty} \frac{o(2^n)}{2^n}, \quad \sum_{n=1}^{\infty} \frac{e(2^n)}{2^n}$$

where $o(n)$ ($e(n)$) count the number of **odd (even) digits** in n . Thus $o(901) = 2, e(901) = 1, o(811) = 2$.

9. Determine the behaviour of the dynamical system

$$(x, y) \mapsto (y, x^2 - y^2) \text{ as } (x_0, y_0) \text{ ranges over } \mathbb{R}^2.$$

10. Minimize

$$\exp(\sin(50x)) + \sin(60e^y) + \sin(70 \sin x) + \sin(\sin(80y)) - \sin(10(x + y)) + (x^2 + y^2)/4$$

