Five Abstracts Follow

http://www.carma.newcastle.edu.au/jon/Thai-Abstracts.pdf

Exploratory Experimentation in Mathematics

Jonathan M. Borwein FRSC, FAAS, BAS, FAA*

March 18, 2013

Abstract. The mathematical research community is facing a great challenge to re-evaluate the role of proof in light of the growing power of cloud computing, of current computer systems, of modern mathematical computing packages, and of the growing capacity to data-mine on the Internet. Add to that the enormous complexity of many modern capstone results such as the Poincaré conjecture, Fermat's last theorem, and the Classification of finite simple groups.

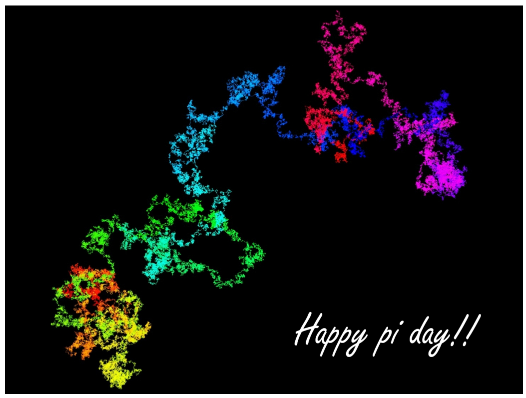
As the need and prospects for inductive mathematics blossom, the requirement to ensure the role of proof is properly founded remains undiminished. I shall look at the philosophical context with examples and then offer some of five bench-marking examples of the opportunities and challenges we face.

This is based on joint work with David Bailey in an eponymous paper which appeared in the 2011 *Notices of the AMS* `http://www.carma.newcastle.edu.au/~jb616/expexp.pdf`.

Seeing things in mathematics by walking on numbers: and some related ideas

Jonathan M. Borwein FRSC, FAAS, BAS, FAA*

March 18, 2013



Abstract. There is a growing need to visualize large mathematical data sets. Motivated by our own requirements—especially in optimization and number theory—we describe and illustrate various tools for representing floating point numbers (or lists) as planar (or three dimensional) walks and for quantitatively measuring their "randomness" or "structure", see [1]. This and much more material is available at the *Walking on Numbers* web page

http://carma.newcastle.edu.au/walks/.

I. The main part of this lecture with all animations embedded is at

http://www.carma.newcastle.edu.au/jon/walking.pdf

and in lower resolution at

http://www.carma.newcastle.edu.au/jon/walking-sm.pdf

II. As time permits we will also illustrate *dynamic geometry tools* for analysis of iterative systems, especially *projection algorithms* [2,3]. See

http://www.carma.newcastle.edu.au/jon/dr-fields11.pptx

and

http://www.carma.newcastle.edu.au/jon/portal.html.

References

[1] F. Aragon, D. H. Bailey, J.M. Borwein and P.B. Borwein, "Walking on real numbers," *Mathematical Intelligencer*, **35** (1) (2013), 42–60. (DOI10.1007/s00283-012-9340-x.)

[2] F. Aragon and J.M. Borwein, "Global convergence of a non-convex Douglas-Rachford iteration." *J. Global Optimization.* E-published July 2012. DOI 10.1007/s10898-012-9958-4.

[3] J.M. Borwein and Brailey Sims, "The Douglas-Rachford algorithm in the absence of convexity." Chapter 6, pp. 93–109 in *Fixed-Point Algorithms for Inverse Problems in Science and Engineering* in *Springer Optimization and Its Applications*, volume 49, 2011.

*Laureate Professor and Director CARMA, University of Newcastle, NSW Australia 2308. Email: jonathan.borwein@newcastle.edu.au, jon.borwein@gmail.com.

Entropy and Projection Methods for (Non-)Convex Programming

Jonathan M. Borwein FRSC, FAAS, BAS, FAA*

March 18, 2013

Abstract.
ABSTRACT. I shall discuss in "tutorial mode" the formalization of inverse problems such as occur in *signal recovery* and *option pricing* as (convex and non-convex) optimization problems over an infinite dimensional space of signals.

I shall touch on the following topics:

1. The impact of the choice of "entropy" (e.g., Boltzmann-Shannon, Burg entropy, Fisher information) on the *well-posedness* of the problem and the form of the solution.

2. Convex programming duality: what it is and what it buys you.

3. Consequences for algorithm design.

4. Non-convex extensions: life is often hard but not always. Some things work that really should not!

REFERENCES.

[1] J.M. Borwein and A.S. Lewis, *Convex Analysis and Nonlinear Optimization. Theory and Examples*, Canadian Mathematical Society Books in Math, Volume 3, Springer-Verlag, New York, 2000. Revised edition 2005.
[2] J.M. Borwein and Qiji Zhu, *Techniques of Variational Analysis and Nonlinear Optimization*, Canadian Mathematical Society Books in Math, Volume 20, Springer-Verlag, New York, 2005.
[3] J.M. Borwein and J. Vanderwerff, *Convex Functions: Constructions, Characterizations and Counterexamples*, Encyclopedia of Mathematics and Applications, **109**, Cambridge University Press, 2010.

Related papers are available on the CARMA document server
http://docserver.carma.newcastle.edu.au/ and on my web pages
http://www.carma.newcastle.edu.au/jon

Maximizing Surprise

Jonathan M. Borwein FRSC, FAAS, BAS, FAA*

March 18, 2013

ABSTRACT. The *Surprise Examination* or *Unexpected Hang- ing Paradox* has long fascinated mathematicians and philosophers, as the number of publications devoted to it attests. For an exhaustive bibliography on the subject, the reader is referred to Tim Chow's article http://www-math.mit.edu/~tchow/unexpected.pdf.

Herein, the optimization problems arising from an information-theoretic or 'entropic' avoidance of the Paradox are examined and solved. They provide a very satisfactory application of both the Kuhn-Tucker theory and of various classical inequalities and estimation techniques.

Although the necessary convex analytic concepts are recalled in the course of the presentation, some elementary knowledge of optimization is assumed. Those without this background may simply skip a couple of proofs and few technical details.

REFERENCES.

1. D. Borwein, J. M. Borwein and P. Maréchal, "Surprise maximization," *American Math. Monthly*, **107** June-July 2000, 527–537.

2. Related papers are available on the CARMA document server http://docserver.carma.newcastle.edu.au/ and on my web pages http://www.carma.newcastle.edu.au/jon

Abstract Best Approximation in (Reflexive) Banach Space

Jonathan M. Borwein FRSC, FAAS, BAS, FAA*

March 18, 2013

Abstract. In **Part I** and **Part II** of these lectures I shall sketch some of the key results known about existence of points with *best approximations* in Banach space.

That is, what can we say about *nearest points*: points x which are members of

$$\operatorname{argmin}_{a \in A} \|x - a\|$$

when A is a non-trivial norm-closed set in a Banach space $(X, \|\cdot\|)$?

Convex and nonsmooth analysis, renorming theory and Banach space geometry theory provide crucial tools. We shall explore all of these. My main source is [1] which, while twenty five years old has largely, not been superseded—but see [2]. Porosity is an exception—see [3,4].

In **Part III** I shall discuss the *Cebysev problem* relying mainly on [5].

Recall that a set S is *Cebysev* if every point in the space X has a *unique nearest point* in the set S. Klee in 1962 asked

Is every Cebysev set in Hilbert space convex (and closed)?

In Euclidean space the answer given by the *Motzkin-Bunt theorem* is 'yes'. I shall describe four proofs methods in detail in [6, §9.2]. They rely respectively on

- topological fixed point theory;
- nonsmooth variational analysis;
- conjugate convex duality;
- inversive geometry;

I shall then discuss the situation in Hilbert space where the problem is still open. Each of the four proofs offers different insight into the issues.

In **Part IV** I shall discuss more generally five notions of *small sets in Banach spaces*. All are closed under *translation, countable union and inclusion* and so can be used to do infinite dimensional analysis even though no *Haar measure* exists. I shall also touch on how they apply to (i) differentiability of convex or Lipschitz functions and (ii) best approximation, see [7, §4.6] and [8].

References

[1] J.M. Borwein and S. Fitzpatrick, “Existence of nearest points in Banach spaces, *Canadian Journal of Mathematics*, **61** (1989), 702–720.

[2] Stefan Cobzas, “Geometric properties of Banach spaces and the existence of nearest and farthest points,*Abstr. Appl. Anal.*, **3** (2005),259-285.

[3] P. Revalski and N.V. Zhivkov: “Small sets in best approximation theory.*J. Global Opt*, **50**(1) (2011), 77–91.

[4] P. Revalski and N.V. Zhivkov: “Best approximation problems in compactly uniformly rotund spaces, *J. Convex Analysis*, **19**(4) (2012), 1153–1166.

[5] J.M. Borwein, “Proximality and Chebyshev sets, *Optimization Letters*, **1**(1) (2007), 21–32.

[6] J.M. Borwein and A.S. Lewis, *Convex Analysis and Nonlinear Optimization. Theory and Examples*, CMS Springer-Verlag, Second extended edition, 2005. Paperback, 2009.

[7] J.M. Borwein and J. Vanderwerff, *Convex Functions: Constructions, Characterizations and Counterexamples*, Encyclopedia of Mathematics and its Applications, **109**, Cambridge University Press, 2010.

[8] J. Lindenstrauss, D. Preiss and J. Tisier, *Fréchet Differentiability of Lipschitz Functions and Porous Sets in Banach Spaces* (AM-179), Princeton University Press, 2012.