SEVEN LECTURES on

Variational Analysis

Generalized Nash Equilibrium Problems, Bilevel programming and MPEC

CIMPA-UNESCO-MESR-MINECO-INDIA

New Delhi, India, November 25-December 6

Jonathan Borwein FRSC FAA FAAS http://www.carma.newcastle.edu.au/jon/

Laureate Professor University of Newcastle, NSW

Director, Centre for **Computer Assisted Research Mathematics and Applications**

Revised 16-11-2013

Seven Lectures on VA

Introduction to variational analysis. *I will lecture (primarily) out of*

J.M. Borwein and Qiji Zhu, Techniques of Variational Analysis,
CMS Books, volume 20, Springer-Verlag, New York, May 2005.
ISBN: 0387242988. Paperback, 2010.

Lecture 1 and 2: Introduction to Variational analysis and Variational principles. *(Ch 1 and Ch 2)*

Lecture 3 and 4: *Nonsmooth analysis: normal cones and subdifferentials of lsc functions, Fréchet and limiting calculus, coderivatives and their calculus rules. (Ch 3 and Ch 4)*

Lecture 5 and 6*: Multifunction analysis: sequences of sets, continuity of maps, minimality. (Ch 4 and Ch 5)*

Lecture 7*: Applications to distance functions, convex analysis and monotone operators. (Ch 5)*

Seven Lectures on VA

J.M. Borwein and Q.J. Zhu
Techniques of Variational Analysis

Ouvrages de mathématiques de la SMC

Variational arguments are classical techniques whose use can be traced back to the early development of the calculus of variations and further. Rooted in the physical principle of least action they have wide applications in diverse fields. This book provides a concise account of the essential tools of infinite-dimensional first-order variational analysis illustrated by applications in many areas of analysis, optimization and approximation, dynamical systems, mathematical economics and elsewhere. The book is aimed at both graduate students in the field of variational analysis and researchers who use variational techniques, or think they might like to. Large numbers of (guided) exercises are provided that either give useful generalizations of the main text or illustrate significant relationships with other results.

Jonathan M. Borwein, FRSC is Canada Research Chair in Collaborative Technology at Dalhousie University. He received his Doctorate from Oxford in 1974 and has been on faculty at Waterloo, Carnegie Mellon and Simon Fraser Universities. He has published extensively in optimization, analysis and computational mathematics and has received various prizes both for research and for exposition.

Qiji J. Zhu is a Professor in the Department of Mathematics at Western Michigan University. He received his doctorate at Northeastern University in 1992. He has been a Research Associate at University of Montreal, Simon Fraser University and University of Victoria, Canada.

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Techniques of Variational Analysis

CMS Books in Mathematics

J.M. Borwein
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Techniques of Variational Analysis

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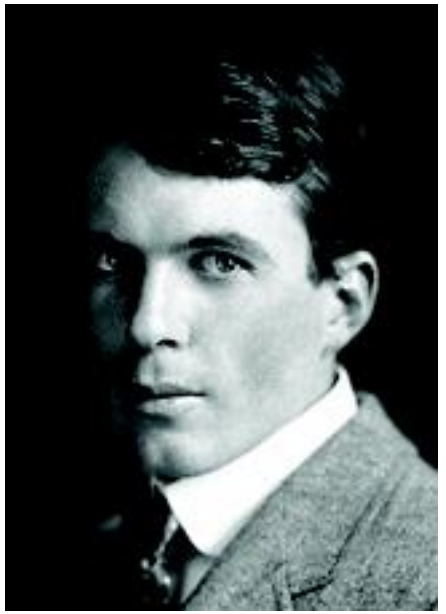
I shall follow Sir Lawrence Bragg (1890-1971)



"I feel so strongly about the wrongness of reading a lecture that my language may seem immoderate.

…

The spoken word and the written word are quite different arts.

…

I feel that to collect an audience and then read one's material is like inviting a friend to go for a walk and asking him not to mind if you go alongside him in your car."

ABSTRACT

Techniques of Variational Analysis

- ✓ I intend to show you the general **patterns**, flavours of proofs and some of the applications of modern (convex, smooth and nonsmooth) variational analysis.
- ✓ You will see bits and pieces in the other lectures (sometimes before and sometimes after I lecture on the topic).
- ✓ The whole text has been converted to a **lecture presentation**: download here.
- ✓ So I expect you to read ahead (or afterwards) and find more details there ….

ABSTRACT

Techniques of Variational Analysis

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Links

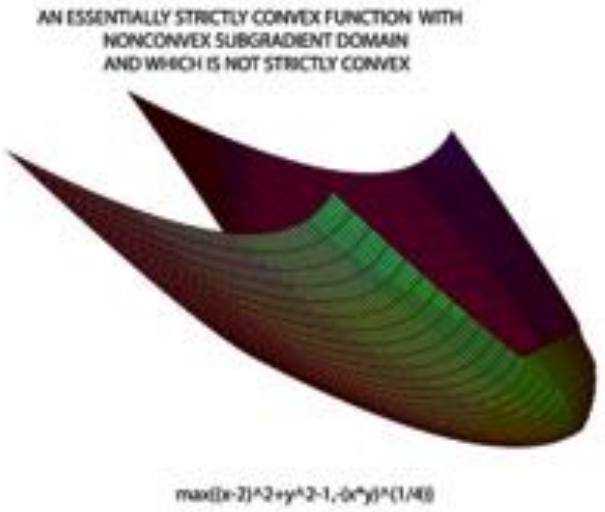
CMS Publications

Amazon

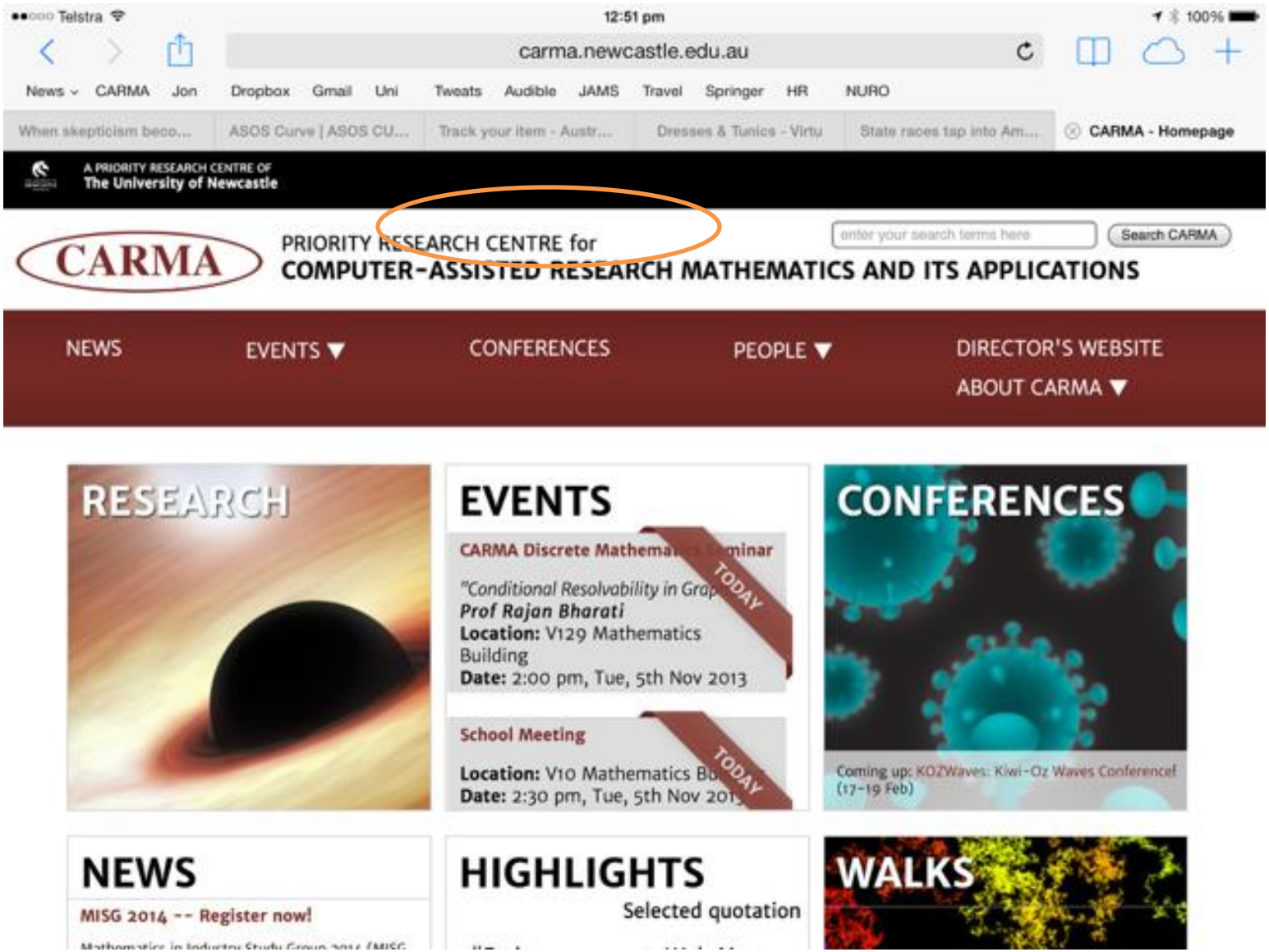
Springer's CMS Book Site

Other

- The textbook as Lectures and Abstract
- Website for my 2010 book on Convex Functions
  (*Cambridge University Press*, 2010)
- Reviews for my book Convex Analysis and Nonlinear Optimization
  (*Springer-Verlag*, 2000/05)
- Variational methods in the presence of Symmetry
  (*Advances in Nonlinear Analysis*, 2013)
- Jean-Paul Penot's book Calculus without derivatives
  (Springer *Graduate Texts in Mathematics, Vol. 942*, 2013)
- Winfried Schirotzek's book Nonsmooth Analysis
  (*Springer Universitext*, 2007)
- My other current CARMA research



ABSTRACT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Prof Rajan Bharati
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Date: 2:00 pm, Tue, 5th Nov 2013
School Meeting
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Location: V10 Mathematics Bu
Date: 2:30 pm, Tue, 5th Nov 201
CONFERENCES
Coming up: KOZWaves: Kiwi-Oz Waves Conference!
(17-19 Feb)
NEWS
MISG 2014 -- Register now!
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Jim (Qiji) Zhu

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Lecture 1 and 2: Introduction to **Variational analysis** and **Variational principles**. *(Ch 1 and Ch 2) [November 25th and 26th]*

Lecture 3 and 4: ***Nonsmooth analysis****: normal cones and subdifferentials of lsc functions, Fréchet and limiting calculus, coderivatives and their calculus rules. (Ch 3 and Ch 4) [26th and 27th]*

Lecture 5 and 6*:* ***Multifunction analysis****: sequences of sets, continuity of maps, minimality. (Ch 4 and Ch 5) [28th and 29th]*

Lecture 7*:* ***Applications*** *to distance functions, convex analysis and monotone operators. (Ch 5) [November 30th]*

SEVEN LECTURES ON VA

Lecture 1 and 2: Introduction to **Variational analysis** and **Variational principles**. *(Ch 1 and Ch 2)*

[Lecture 1. 25/11 10.00-11.00]

[Lecture 2. 26/11 10.00-11.00]

SEVEN LECTURES ON VA

Lecture 3 and 4: ***Nonsmooth analysis****: normal cones and subdifferentials of lsc functions, Fréchet and limiting calculus, coderivatives and their calculus rules. (Ch 3 and Ch 4)*

[Lecture 3. 26/11 13.45-14.45]

[Lecture 4. 27/11 11.15-12.15]

SEVEN LECTURES ON VA

***Lecture 5 and 6**: Multifunction analysis: sequences of sets, continuity of maps, minimality. (Ch 4 and Ch 5)*

[Lecture 5. 28/11 9.00-10.00]

[Lecture 6. 29/11 14.45-15.45]

SEVEN LECTURES ON VA

*Lecture 7: "**Applications**" to distance functions, convex analysis and monotone operators. (Ch 5)*

[Lecture 7. 30/11 9.00-10.00]

SEVEN LECTURES ON VA

The end *"Regeneration"*