Possibilities, Challenges and the Future of Remote Collaboration

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``User-interface criticism is a genre to watch. It will probably be more influential and beneficial to the next century than film criticism was to the twentieth century. The twenty-first century will be filled with surprises, but one can safely count on it to bring more complexity to almost everything. Bearing the full brunt of that complexity, the great user-interface designers of the future will provide people with the means to understand and enrich their own humanity, and to stay human." Jaron Lanier, 1999

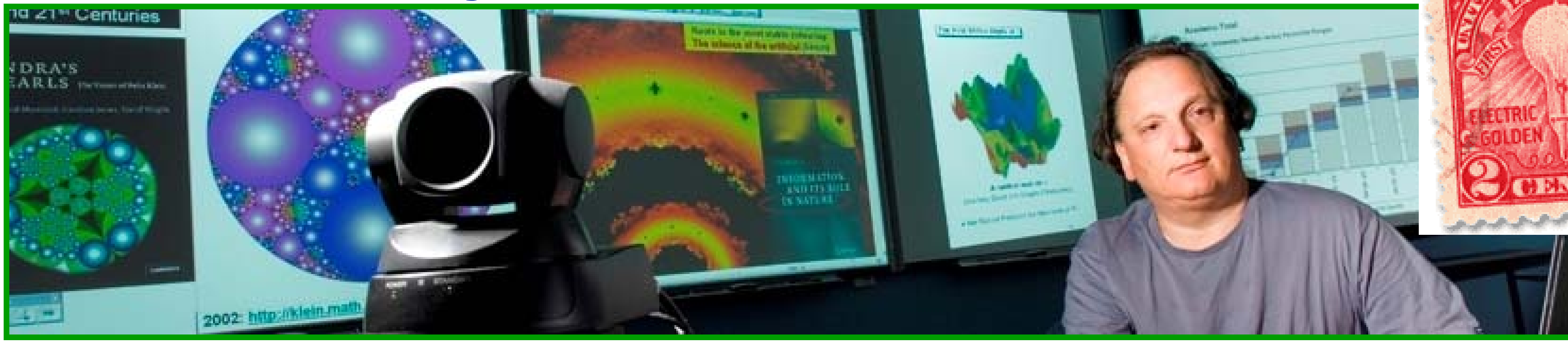
THE UNIVERSITY OF NEWCASTLE AUSTRALIA

Revised 21-04-09

Remote Collaboration or (Anti)-Social Networking?

"On the Internet, nobody knows you're a dog."

G'day from the Newcastle AGR



ABSTRACT The mathematical community (appropriately defined) is facing a great challenge to re-evaluate the role of proof in light of the power of current computer systems, of modern mathematical computing packages and of the growing capacity to data-mine on the internet --- all predicated on ample, robust bandwidth and interoperability.

With great challenges come great opportunities. I intend to discuss the current challenges and opportunities for the collaborative learning and doing of mathematics.

"All truths are easy to understand once they are discovered; the point is to discover them." – Galileo Galilei

Webster (I)

- re-mote *adjective*

Etymology: M. E., from Latin *remotus,* past participle of *removēre* to remove Date: 15th century

1**: separated by an interval or space greater than usual**
an involucre *remote* from the flower

2**: far removed in space, time, or relation:** divergent
the *remote* past, comments *remote* from the truth

3**:** out-of-the-way, secluded a *remote* cabin in the hills

4**: acting, acted on, or controlled indirectly or from a distance**
remote computer operation

also: **relating to the acquisition of information about a distant object** (as by radar or photography) without coming into physical contact with it *remote* sensing

5**:** not arising from a primary or proximate action

Webster (II)

- <u>col·lab·o·rate</u>, *v.i.*

Etymology: Late Latin *collaboratus,* past participle of *collaborare* to labour together, from Latin *com-* + *laborare* to labour Date: 1871

1a: to work, one with another; cooperate, as on a literary work: *They collaborated on a novel*

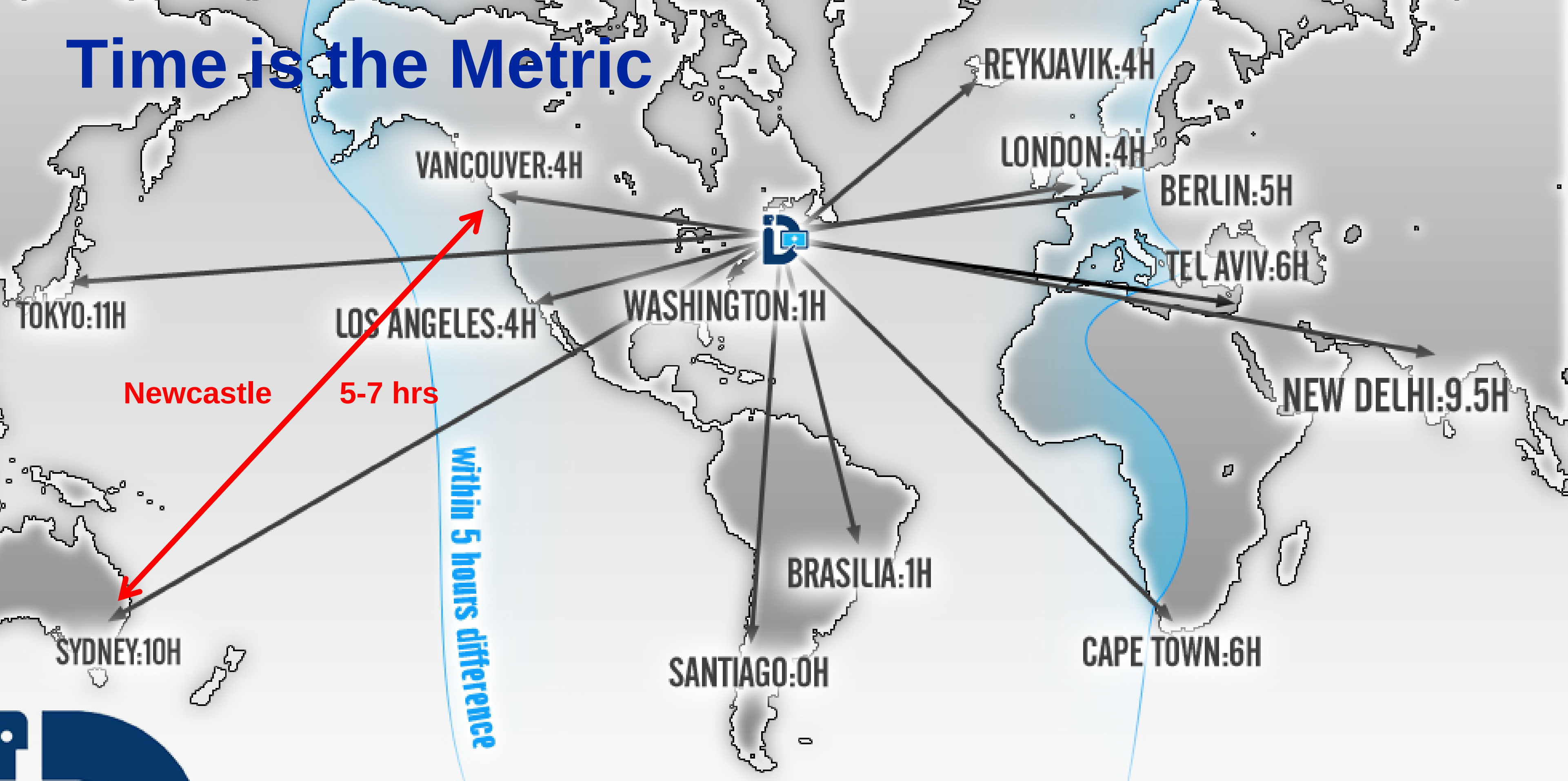
1b: to work jointly with others or together especially in an intellectual endeavour

2**:** to cooperate with or willingly assist an enemy of one's country and especially an occupying force

3: to cooperate with an agency or instrumentality with which one is not immediately connected

<u>col·lab·o·ra·tion</u> *noun* <u>col·lab·o·ra·tive</u> *adjective or noun*

Time is the Metric



D-Drive's Nova Scotia location lends us unusual freedom when interacting globally. Many cities around the world are close enough in a chronological sense to comfortably accommodate real-time collaboration.

Drive

Dalhousie Distributed Research Institute and Virtual Environment

Better Science is the Goal

- ~ 65% of journal literature is digitized
- Search on math
- Math OCR

are coming (slowly)

Best Current Practices: Recommendations on Electronic Information Communication (2004)

Revised version of 2002 Recommendations
Endorsed by the IMU Executive Committee on April 24, 2004
at its 72nd session in Oxford, England

Communication of mathematical research and scholarship is undergoing profound change as new technology creates new ways to disseminate and access the literature. More than technology is changing, however, the culture and practices of those who create, disseminate, and archive the mathematical literature are changing as well. For the sake of present and future mathematicians, we should shape those changes to make them suit the needs of the discipline.

For this reason, we have identified a number of *best practices* for those involved with the mathematical literature – mathematicians, librarians, and publishers. Many of these are practices that apply to other academic disciplines as well. Although we focus primarily on mathematics, we recognize that we can learn from each other as we move forward, and that no single discipline should act in isolation.

Our advice is meant to guide practice as it changes rather than to set forth a collection of firm rules and admonitions. The recommendations concern all forms of scholarly publishing and do not promote any particular form. Indeed, the authors of this document hold many differing views on the future of scholarly publishing. The common principle used to formulate our recommendations is that those who write, disseminate, and store mathematical literature should act in ways that serve the interests of mathematics, first and foremost.

There is little literature on math collaboration

See also the References

Friend or Foe? Examining CAS Use in Mathematics Research

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ABSTRACT

Computer Algebra Systems (CAS) provide sophisticated functionality to assist with mathematical problem solving. Despite their widespread adoption, however, little work in the HCI community has examined the extent to which these computational tools support domain experts. In this paper, we report findings from a qualitative study investigating the work practices and tools of nine mathematicians in a research setting. Counter to our expectations, our data suggests that computational tools play only a minor role in their workflow, with the limited use of CAS owing primarily to four factors: (1) *the need for transparency* in CAS's reasoning to explain computed results; (2) *the problem of rigidity and formality* in CAS's input/output style dialogue; (3) *the need for 2D input* to support a wide range of annotations, diagrams, and in-place manipulation of objects of interest; and (4) *the need for collaboration*, particularly in early stages of problem solving. While grounded in the study of mathematicians, these findings (particularly the first) have implications for the design of computational systems intended to support complex problem solving.

Despite the popularity of these systems, little HCI research has examined these tools in *professional* contexts. Instead, existing research has typically studied this software in educational settings (e.g., [16], [3], [15], [14], [10]), or has focused on specific features in laboratory situations (e.g., the usability of mechanisms to input mathematical expressions [2], [7]). While this past research provides valuable perspectives on this class of software, there is still a need to examine these systems in the context of professional use, where the tools have the potential to impact cognitively demanding tasks in a significant way. Designing tools that augment human intellect remains a challenging endeavor, but must begin with a thorough understanding of the context of use. In this case, there is a need to understand how mathematicians work, as well as how current computational tools integrate with practices, to understand the specific HCI challenges for this class of software.

This paper reports the results of a qualitative study examining the practices, tools, and artifacts of nine expert mathematicians from a research university. We performed semi--structured interviews in the mathematicians' places of work

The Talk (I)

I. Where We Are

- Canada: C2C, Compute Canada, IRMACS
- Australia: AMSI AGR's, shared courses, ANZIAM OCG,
  - $42bn 100Mb National Broadband Network (?)
- Elsewhere: UK, Chile, Skype-Google-PDF annotator etc
  - See IMUonWeb (#29)

II. Where We Want To Be

- Ideally: seamless, integrated, 24/7
  - **complexity vs. compatibility**: spontaneity and preparation
  - common CAS syntax (for DLMF), INTERGEO, cloud tools
- Realistically: many organizational/cultural impediments
  - **synchronicity**: Today, Dal. PhD, IRMACS-Fields Workshop
  - **enterprise IT models**: Columbia interview, Melbourne charges

The Dream of Interoperability

... la plus ça change ...

"Hello, Bob? It's your father again.
I have another question about my new computer.
Can I tape a movie from cable TV then fax it from my VCR to my CD-ROM then E-mail it to my brother's cellular phone so he can make a copy on his neighbor's camcorder?"

The Talk (II)

- **III. Two Mathematical Examples**
  - Digitally-assisted math: What's that number?
  - A dynamic system: Visualization and proof

- **IV. Conclusions and Questions**
  - What do we value?
  - What adds value?
  - What can we afford?
    - — in time, money and effort
    - — for which purposes?

III. What is Digital Assistance?

- Use of Modern Mathematical Computer Packages
  - Symbolic, Numeric, Geometric, Graphical, …
- Use of More Specialist Packages or General Purpose Languages
  - Fortran, C++, CPLEX, GAP, PARI, MAGMA, …
- Use of Web Applications
  - Sloane’s Encyclopedia, Inverse Symbolic Calculator, Fractal Explorer, Euclid in Java, …
- Use of Web Databases
  - Google, MathSciNet, ArXiv, JSTOR, Wikipedia, MathWorld, Planet Math, DLMF, MacTutor, Amazon, …
- All entail **data-mining** [“exploratory experimentation” and “widening technology” as in pharmacology, astrophysics, biotech… (Franklin)]
  - Clearly the boundaries are blurred and getting blurrier

“Knowing things is very 20th century. You just need to be able to find things.”

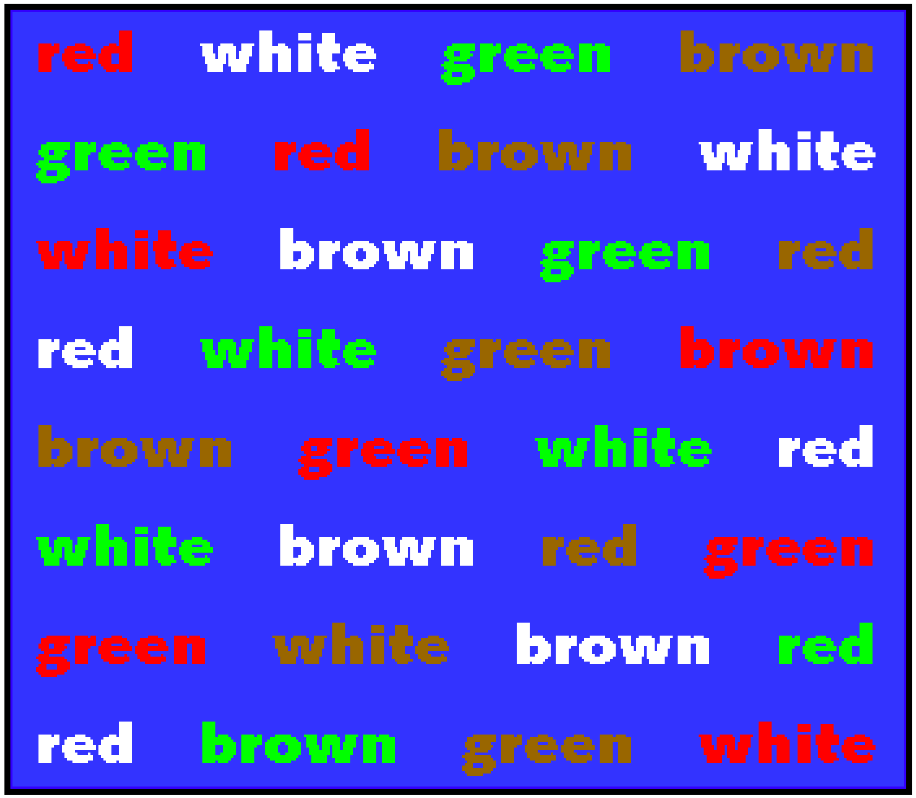
- Danny Hillis

- on how Google has already **changed how we think** in Achenblog, July 1 2008

- changing cognitive styles

Changing User Experience and Expectations

What is attention? (**Stroop** test, 1935)



1. Say the **color** represented by the **word**.
2. Say the **color** represented by the **font** color.

High multitaskers perform # 2 very easily. They are great at suppressing information.

http://www.snre.umich.edu/eplab/demos/st0/stroop_program/stroopgraphicnonshockwave.gif

Acknowledgements: Cliff Nass, CHIME lab, Stanford (interference and twitter?)

Example 1. What's that number? (1995 to 2008)

In **1995** or so Andrew Granville emailed me the number

$$\alpha := 1.4331274267223\ldots$$

and challenged me to identify it (our inverse calculator was new in those days).

I asked for its continued fraction? It was

$$[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, \ldots] \qquad (1)$$

I reached for a good book on continued fractions and found the answer

$$\alpha = \frac{I_0(2)}{I_1(2)}$$

where I_0 and I_1 are Bessel functions of the first kind. (Actually I knew that all arithmetic continued fractions arise in such fashion.)

In 2008 there are at least two or three other strategies:

- Given (1), type "arithmetic progression", "continued fraction" into Google
- Type 1,4,3,3,1,2,7,4,2 into Sloane's Encyclopaedia of Integer Sequences

I illustrate the results on the next two slides:

“arithmetic progression”, “continued fraction”

In Google on October 15 2008 the first three hits were

***Continued Fraction* Constant -- from Wolfram MathWorld**

- 3 visits - 14/09/07Perron (1954-57) discusses *continued fractions* having terms even more general than the *arithmetic progression* and relates them to various special functions. **...**
*mathworld.wolfram.com/***ContinuedFraction***Constant.html - 31k*

HAKMEM -- *CONTINUED FRACTIONS* -- DRAFT, NOT YET PROOFED

The value of a *continued fraction* with partial quotients increasing in *arithmetic progression* is I (2/D) A/D [A+D, A+2D, A+3D, . **...**
www.inwap.com/pdp10/hbaker/hakmem/cf.html - 25k -

On simple *continued fractions* with partial quotients in *arithmetic* ...

0. This means that the sequence of partial quotients of the *continued fractions* under. investigation consists of finitely many *arithmetic progressions* (with **...**
www.springerlink.com/index/C0VXH713662G1815.pdf - by P Bundschuh – 1998

Moreover the MathWorld entry includes

$$[A+D, A+2D, A+3D, \ldots] = \frac{I_{A/D}\left(\frac{2}{D}\right)}{I_{1+A/D}\left(\frac{2}{D}\right)}$$

(Schroeppel 1972) for real A and $D \neq 0$.

Example 1: In the Integer Sequence Data Base

Greetings from The On-Line Encyclopedia of Integer Sequences!

1,4,3,3,1,2,7,4,2

Search

Hints

Search: 1,4,3,3,1,2,7,4,2

Displaying 1-1 of 1 results found. page

Format: long | short | internal | text   Sort: relevance | references | number   Highlight: on | off

A060997   Decimal representation of continued fraction 1, 2, 3, 4, 5, 6, 7, ...

1, 4, 3, 3, 1, 2, 7, 4, 2, 6, 7, 2, 2, 3, 1, 1, 7, 5, 8, 3, 1, 7, 1, 8, 3, 4, 5, 5, 7, 7, 5, 9, 9, 1, 8, 2, 0, 4, 3, 1, 5, 1, 2, 7, 6, 7, 9, 0, 5, 9, 8, 0, 5, 2, 3, 4, 3, 4, 4, 2, 8, 6, 3, 6, 3, 9, 4, 3, 0, 9, 1, 8, 3, 2, 5, 4, 1, 7, 2, 9, 0, 0, 1, 3, 6, 5, 0, 3, 7, 2, 6, 4, 3, 5, 7, 8, 6, 1, 1, 4, 6, 5, 9, 5, 0 (list; cons; graph; listen)

OFFSET	1,2
COMMENT	The value of this continued fraction is the ratio of two Bessel functions: BesselI(0,2)/BesselI(1,2) = A070910/A096789. Or, equivalently, to the ratio of the sums: sum_{n=0..inf} 1/(n!n!) and sum_{n=0..inf} n/(n!n!). - Mark Hudson (mrmarkhudson(AT)hotmail.com), Jan 31 2003
FORMULA	1/A052119.
EXAMPLE	C=1.433127426722311758317183455775 ...
MATHEMATICA	RealDigits[FromContinuedFraction[Range[44]], 10, 110] [[1]] (* Or *) RealDigits[BesselI[0, 2] / BesselI[1, 2], 10, 110] [[1]] (* Or *) RealDigits[Sum[1/(n!n!), {n, 0, Infinity}] / Sum[n/(n!n!), {n, 0, Infinity}], 10, 110] [[1]]
CROSSREFS	Cf. A052119, A001053. Adjacent sequences: A060994 A060995 A060996 this_sequence A060998 A060999 A061000 Sequence in context: A016699 A060373 A090280 this_sequence A129624 A019975 A073871
KEYWORD	cons,easy,nonn
AUTHOR	Robert G. Wilson v (rgwv(AT)rgwv.com), May 14 2001

The **Inverse Calculator** returns

Best guess:

BesI(0,2)/BesI(1,2)

- We show the ISC on another number next
- Most functionality of ISC is built into "identify" in Maple

"The price of metaphor is eternal vigilance." - Arturo Rosenblueth & Norbert Wiener quoted by R. C. Leowontin, *Science* p.1264, Feb 16, 2001 [Human Genome Issue].

The Inverse Symbolic Calculator (ISC) uses a combination of lookup tables and integer relation algorithms in order to associate with a user-defined, truncated decimal expansion (represented as a floating point expression) a closed form representation for the real number.

The ISC in Action

ISC+ inverse symbolic calculator

Standard lookup results for **12.587886229548403854**

exp(1)+Pi^2

ISC [The original ISC]

The Dev Team: Nathan Singer, Andrew Shouldice, Lingyun Ye, Tomas Daske, Peter Dobcsanyi, Dante Manna, O-Yeat Chan, Jon Borwein

3.146264370 [Try it!]

19.99909998 [Try it!]

ISC [The original ISC]

The Dev Team: Nathan Singer, Andrew Shouldice, Lingyun Ye, Tomas Daske, Peter Dobcsanyi, Dante Manna, O-Yeat Chan, Jon Borwein

The ISC presently accepts either floating point expressions or correct Maple syntax as input. However, for Maple syntax requiring too long for evaluation, a timeout has been implemented.

Visit

[Jon Borwein's Webpage]

[David Bailey's Webpage]

[Math Resources Portal]

http://ddrive.cs.dal.ca/~isc

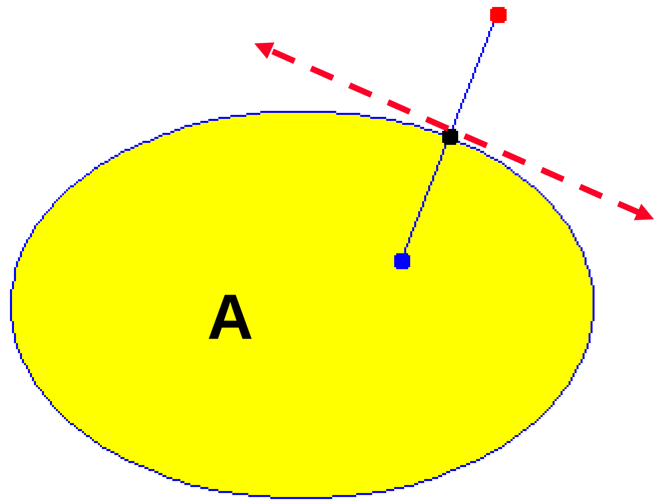
- ISC+ runs on Glooscap
- Less lookup & more algorithms than 1995

Example 2: Phase Reconstruction Models

Projectors and Reflectors: $P_A(x)$ is the metric projection or nearest point and $R_A(x)$ reflects in the tangent: x is red

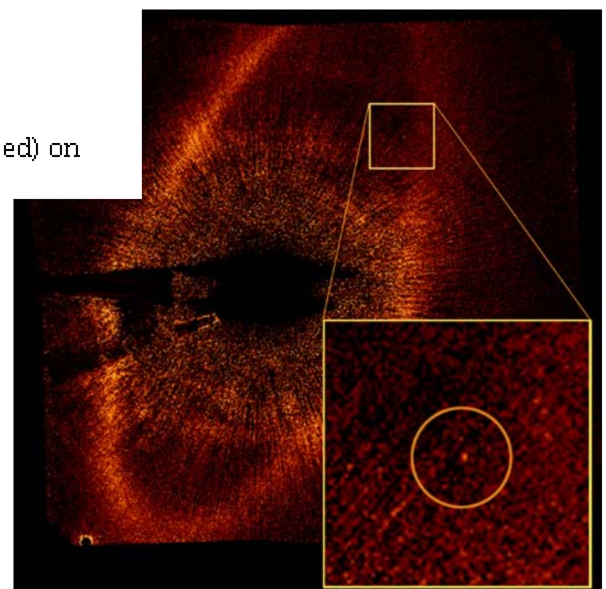
Veit Elser, Ph.D.

Solving Sudoku

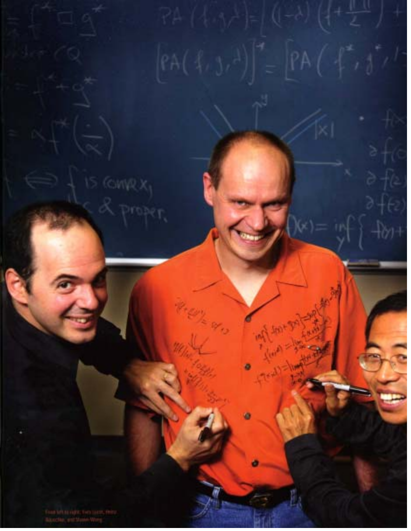


projection (black) and reflection (blue) of point (red) on boundary (blue) of ellipse (yellow)

Finding exoplanet Fomalhaut in Piscis



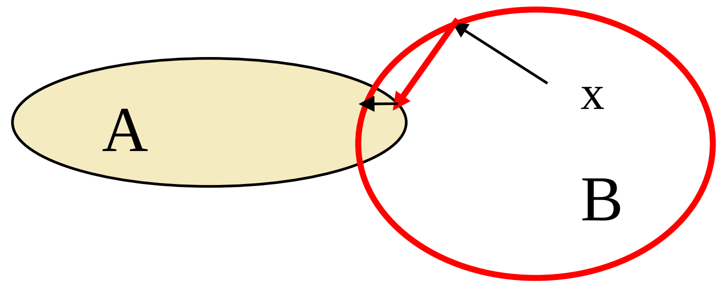
Inverse Problems as Feasibility Problems



OCANA@UBC-O

To find $x \in A \cap B$ one may use the *method of alternating projections*

$$y_n := P_B(x_n), \quad x_{n+1} := P_A(y_n)$$



This parallelizes efficiently to products of sets (project and average):

$$A := \prod_{k=1}^{M} A_k, \qquad B := \{(x_1, x_2, \cdots, x_N) | x_1 = x_2 = \cdots = x_N\}$$

$$P_A = (P_{A_1}, P_{A_2}, \cdots, P_{A_N}), \quad P_B\,(x_1, x_2, \cdots, x_N) = \left(\frac{\sum_{k=1}^{N} x_k}{N}, \cdots, \frac{\sum_{k=1}^{N} x_k}{N} \right).$$

- The theory is rich for convex sets; missing in general. Yet:
- It fixed the abberation problem on Hubble (non-convex).

Example 2: Phase Reconstruction

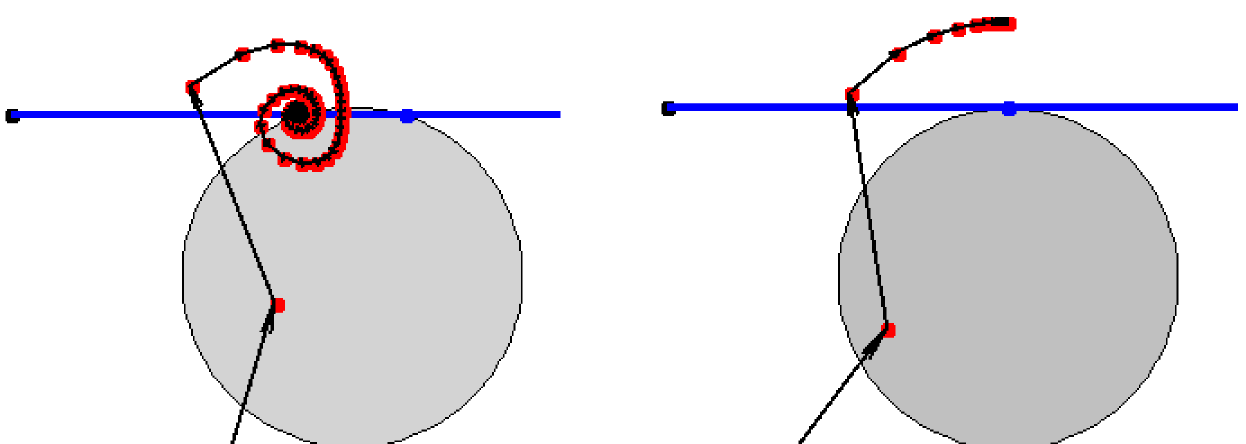
In a wide variety of problems (protein folding, 3SAT, Sudoku) B is non-convex but “divide and concur” works better than theory can explain. It is:

$$R_A(x) := 2\,P_A(x) - x \text{ and } x \to \frac{x+R_A(R_B(x))}{2}$$

Consider the simplest case of a line A of height α and the unit circle B. With $z_n := (x_n, y_n)$ the reflection algorithm becomes

$$x_{n+1} := \cos\theta_n, y_{n+1} := y_n + \alpha - \sin\theta_n, \quad (\theta_n := \arg z_n)$$

For α=0 convergence to one of the two points in A ∩ B iff start off vertical axis (CHAOS on y-axis). **For α>1** (infeasible) iterates go vertically to infinity. **For α=1** (tangent) iterates converge to point above tangent. **For $\alpha \in$ (0,1)** the images are lovely but proofs escape us. Maple and Cinderella pictures follow:



An ideal problem to introduce early under-graduates to research, with many accessible extensions in 2 or 3 dimensions

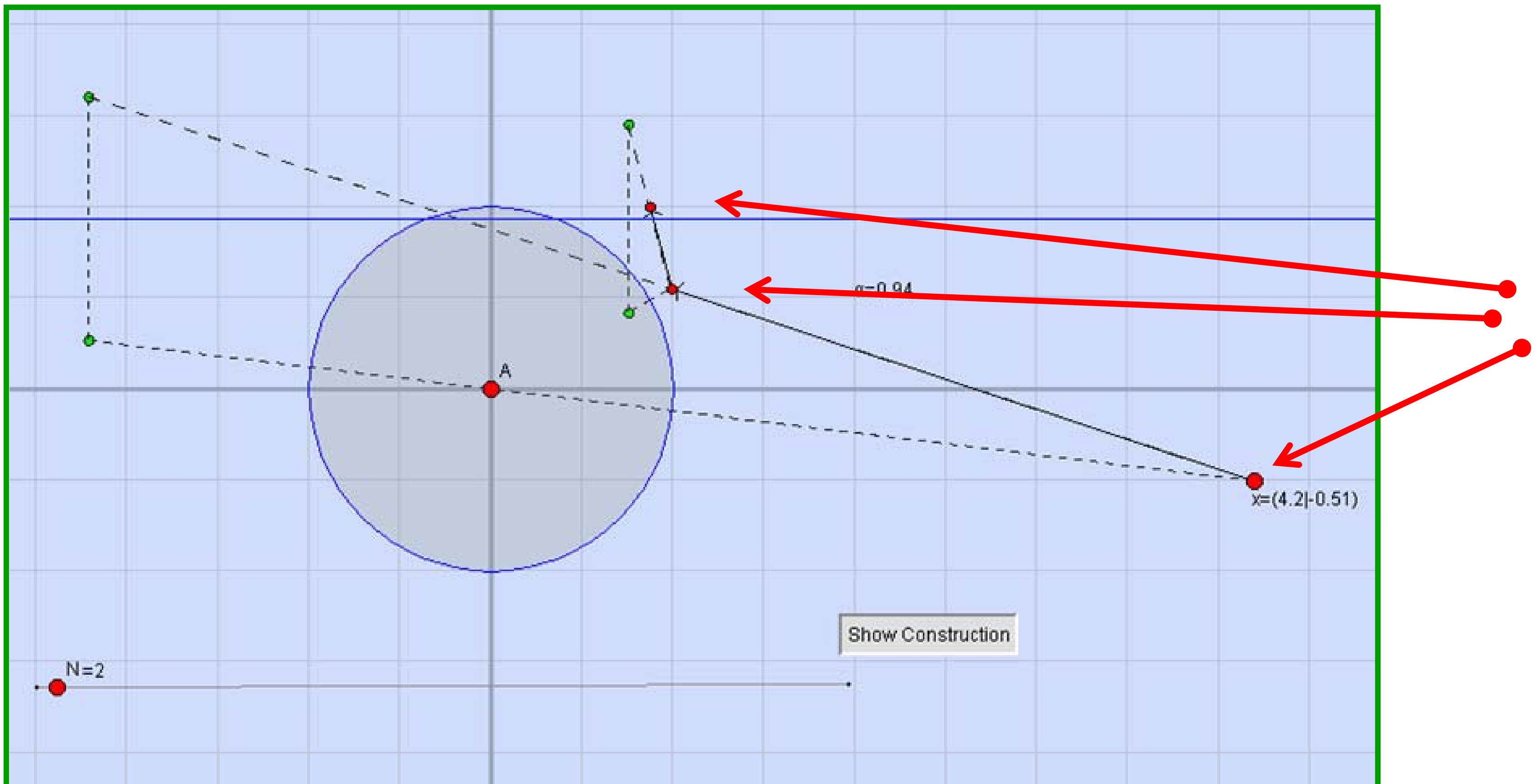
Interactive Phase Recovery in Cinderella

Consider the simplest case of a line A of height α and the unit circle B.

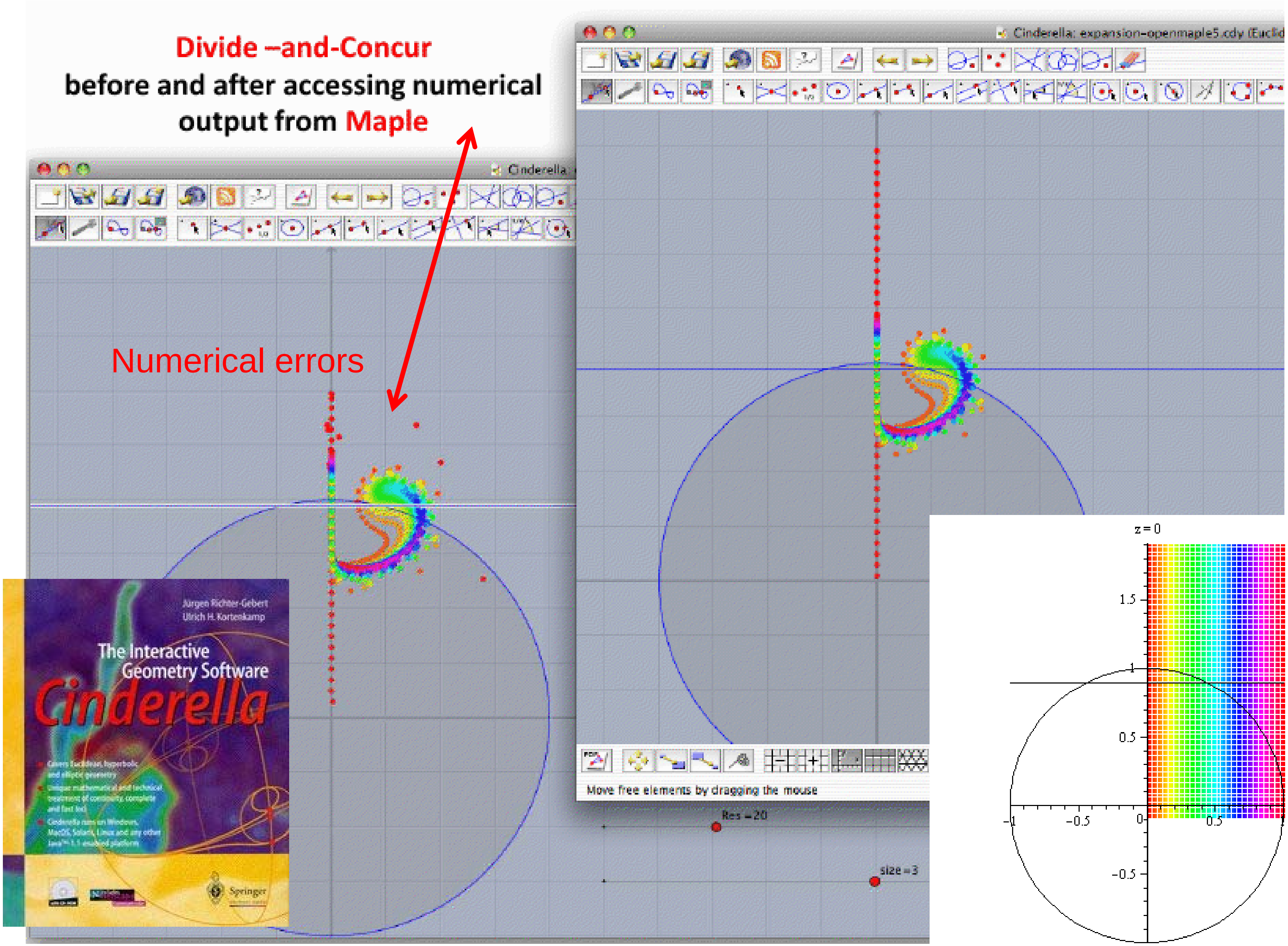
With $z_n := (x_n, y_n)$ the iteration becomes

$$x_{n+1} := \cos\theta_n, y_{n+1} := y_n + \alpha - \sin\theta_n, \quad (\theta_n := \arg z_n)$$

For $\alpha \in (0,1)$ the pictures are lovely but proofs escape me. A *Cinderella* picture of two steps from (4.2,-0.51) follows:



The Grief is in the GUI



A Sad Story (UK)

1. **Teaching Maths In 1970** A logger sells a lorry load of timber for £1000. His cost of production is 4/5 of the selling price. What is his profit?
2. **Teaching Maths In 1980** A logger sells a lorry load of timber for £1000. His cost of production is 4/5 of the selling price, or £800. What is his profit?
3. **Teaching Maths In 1990** A logger sells a lorry load of timber for £1000. His cost of production is £800. Did he make a profit?
4. **Teaching Maths In 2000** A logger sells a lorry load of timber for £1000. His cost of production is £800 and his profit is £200. Underline the number 200.
5. **Teaching Maths In 2008** A logger cuts down a beautiful forest because he is a totally selfish and inconsiderate bastard and cares nothing for the habitat of animals or the preservation of our woodlands. He does this so he can make a profit of £200. What do you think of this way of making a living?

   Topic for class participation after answering the question: How did the birds and squirrels feel as the logger cut down their homes? (There are no wrong answers. If you are upset about the plight of the animals in question counselling will be available.)

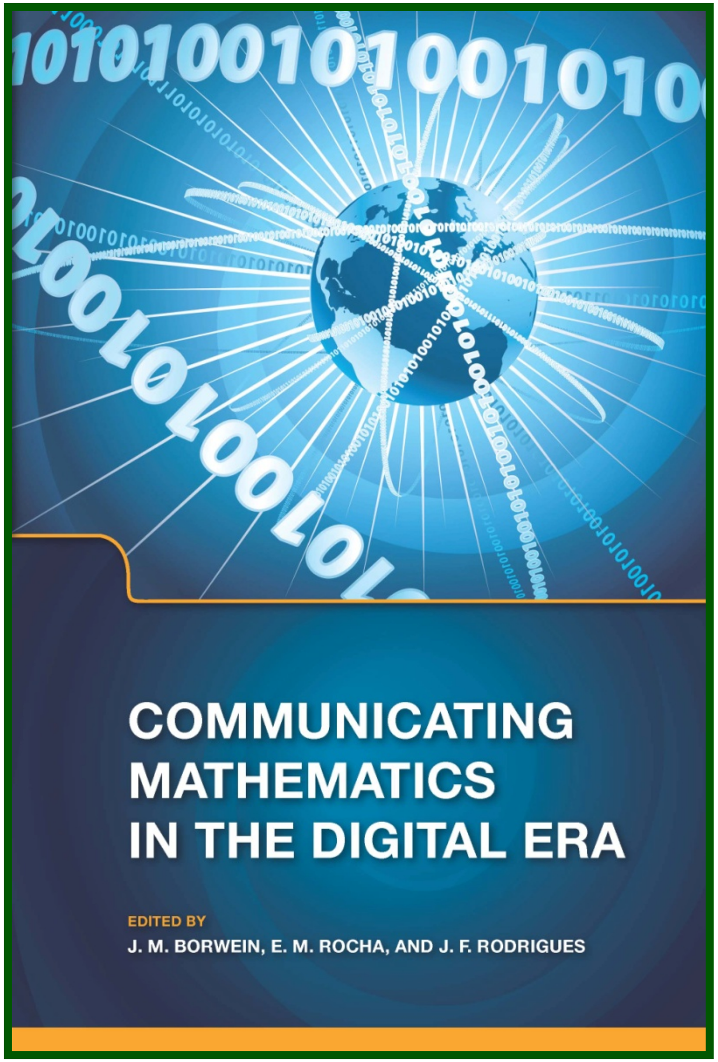
IV. Conclusions

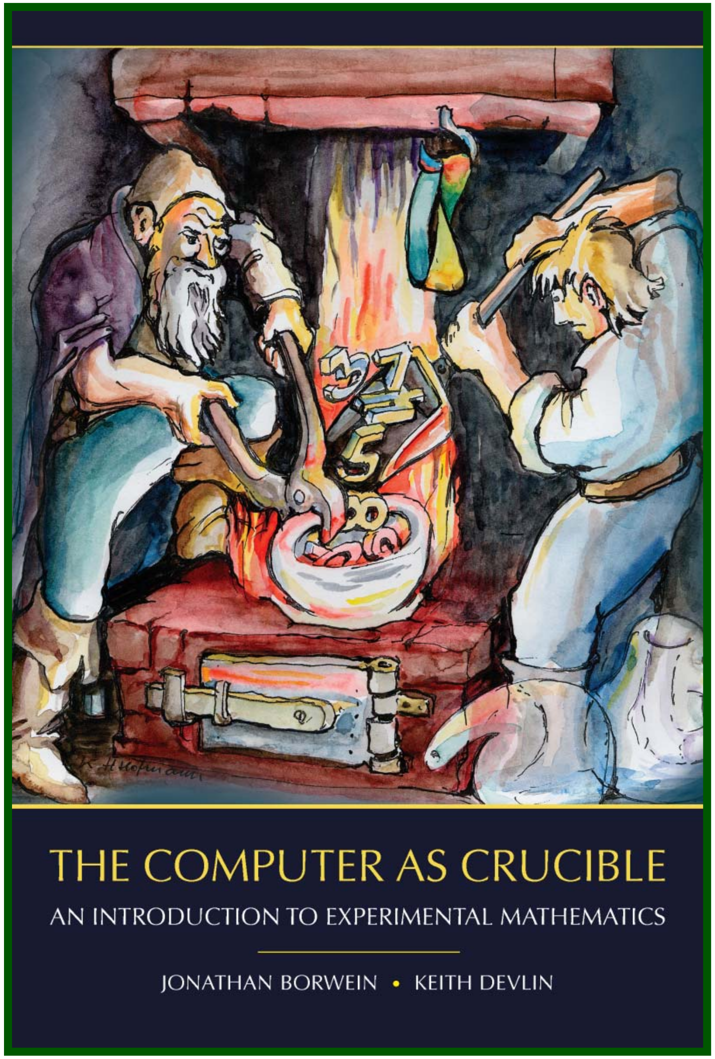
- We like students of **2010** live in an information-rich, judgement-poor world
- The explosion of information is not going to diminish
  - nor is the desire (need?) to collaborate remotely
- So we have to learn and teach judgement (not obsession with plagiarism)
  - that means mastering the sorts of tools I have illustrated
- We also have to acknowledge that most of our classes will contain a very broad variety of skills and interests (few future mathematicians)
  - properly balanced, discovery and proof can live side-by-side and allow for the ordinary and the talented to flourish in their own fashion
- **Impediments** to the assimilation of the tools I have illustrated are myriad
  - as I am only too aware from recent experiences
- These impediments include our own inertia and
  - organizational and technical bottlenecks (IT - not so much dollars)
  - under-prepared or mis-prepared colleagues
  - the dearth of good modern syllabus material and research tools
  - the lack of a compelling business model (societal goods)

"The plural of 'anecdote' is not 'evidence'."
- Alan L. Leshner, *Science*'s publisher

References

Talks on C2C Seminar, Digitally-assisted Mathematics (Part I, Part II), What's New, and Interdisciplinarity.





2008 AKP Peters **books**