CARMA and Me for 18-08-2012 CARMA Annual Retreat

Jonathan M. Borwein FRSC FAA FAAAS

Laureate Professor & Director of CARMA, University of Newcastle
URL: http://carma.newcastle.edu.au/jon/retreat12.pdf NEWS: http://carma.newcastle.edu.au/carmanews.shtml

Priority Research Centre for
Computer **A**ssisted **R**esearch **M**athematics and its **A**pplications

Revised: August 17 2012

Please:

1. Bookmark this Home page
2. Regularly monitor Events
   - and make sure they are advertised
3. Report Issues to
   - David Allingham and Roslyn Hickson
4. Post News Items

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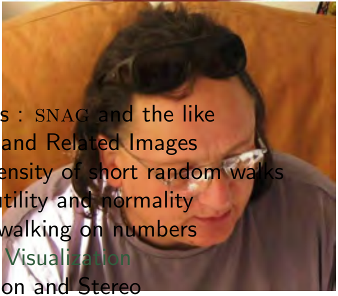
Contents. We will *sample* the following:



Experimental Mathematics: what it is?

Experimental mathematics is the use of a computer to run computations—sometimes no more than trial-and-error tests—to look for patterns, to identify particular numbers and sequences, to gather evidence in support of specific mathematical assertions that may themselves arise by computational means, including search.

Like contemporary chemists — and before them the alchemists of old—who mix various substances together in a crucible and heat them to a high temperature to see what happens, today's experimental mathematicians put a hopefully potent mix of numbers, formulas, and algorithms into a computer in the hope that something of interest emerges. (JMB-Devlin, 2008, p. 1)

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Experimental Mathematics: Integer Relation Methods

Secure Knowledge without Proof. Given real numbers $\beta, \alpha_1, \alpha_2, \ldots, \alpha_n$ Ferguson's integer relation method (PSLQ), finds a nontrivial linear relation of the form

$$a_0\beta + a_1\alpha_1 + a_2\alpha_2 + \cdots + a_n\alpha_n = 0, \qquad (1)$$

where a_i are integers—if one exists and provides an exclusion bound otherwise.

- If $a_0 \neq 0$ then (1) assures β is in rational vector space generated by $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$.
- $\beta = 1, \alpha_i = \alpha^i$ means α is algebraic of degree n
- **2000** *Computing in Science & Engineering*: PSLQ one of top 10 algorithms of 20th century

PROFILE: HELAMAN FERGUSON

Carving His Own Unique Niche, In Symbols and Stone

By refusing to choose between mathematics and art, a self-described "misfit" has found the place where parallel careers meet

CMS D.Borwein Prize

Madelung constant

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Top Ten Algorithms: all but one well used in CARMA

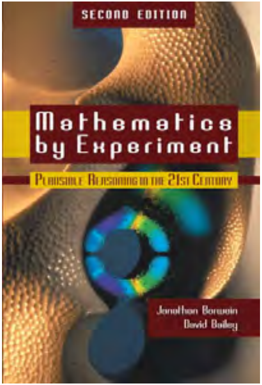
Algorithms for the Ages

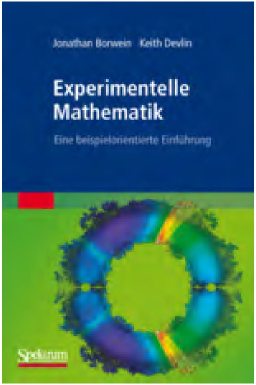
"Great algorithms are the poetry of computation," says Francis Sullivan of the Institute for Defense Analyses' Center for Computing Sciences in Bowie, Maryland. He and Jack Dongarra of the University of Tennessee and Oak Ridge National Laboratory have put together a sampling that might have made Robert Frost beam with pride--had the poet been a computer jock. Their list of 10 algorithms having "the greatest influence on the development and practice of science and engineering in the 20th century" appears in the January/February issue of Computing in Science & Engineering. If you use a computer, some of these algorithms are no doubt crunching your data as you read this. The drum roll, please:

1. **1946: The Metropolis Algorithm for Monte Carlo.** Through the use of random processes, this algorithm offers an efficient way to stumble toward answers to problems that are too complicated to solve exactly.
2. **1947: Simplex Method for Linear Programming.** An elegant solution to a common problem in planning and decision-making.
3. **1950: Krylov Subspace Iteration Method.** A technique for rapidly solving the linear equations that abound in scientific computation.
4. **1951: The Decompositional Approach to Matrix Computations.** A suite of techniques for numerical linear algebra.
5. **1957: The Fortran Optimizing Compiler.** Turns high-level code into efficient computer-readable code.
6. **1959: QR Algorithm for Computing Eigenvalues.** Another crucial matrix operation made swift and practical.
7. **1962: Quicksort Algorithms for Sorting.** For the efficient handling of large databases.
8. **1965: Fast Fourier Transform.** Perhaps the most ubiquitous algorithm in use today, it breaks down waveforms (like sound) into periodic components.
9. **1977: Integer Relation Detection.** A fast method for spotting simple equations satisfied by collections of seemingly unrelated numbers.
10. **1987: Fast Multipole Method.** A breakthrough in dealing with the complexity of n-body calculations, applied in problems ranging from celestial mechanics to protein folding.

From *Random Samples*, Science page 799, February 4, 2000.

Experimental Mathematics: PSLQ is core to CARMA





Experimental Mathematics (2004-08, 2009, 2010)

Notices of AMS 2011: and hundreds of online publications

Exploratory Experimentation and Computation

David H. Bailey and Jonathan M. Borwein

> The authors' thesis—once controversial, but now a commonplace—is that computers can be a useful, even essential, aid to mathematical research.
>
> —Jeff Shallit

Jeff Shallit wrote this in his recent review (MR2427663) of [10]. As we hope to make clear, Shallit was entirely right in that many, if not most, research mathematicians now use the computer in a variety of ways to draw pictures, inspect numerical data, manipulate expressions symbolically, and run simulations. However, it seems to us that there has not yet been substantial and intellectually rigorous progress in the way mathematics is presented in research papers, textbooks, and classroom instruction or in how the mathematical discovery process is organized.

Mathematicians Are Humans

We share with George Pólya (1887–1985) the view [25, vol. 2, p. 128] that, while learned,

> intuition comes to us much earlier and with much less outside influence than formal arguments.

David H. Bailey is Chief Technologist of the Computational Research Department at Lawrence Berkeley National Laboratory. His email is dhbailey@lbl.gov. This work was supported by the director, Office of Computational and Technology Research, Division of Mathematical, Information, and Computational Sciences of the U.S. Department of Energy, under contract number DE-AC02-05CH11231.

Jonathan M. Borwein is Laureate Professor at the Centre for Computer Assisted Research Mathematics and its Applications (CARMA) at the University of Newcastle, Australia. His email address is jonathan.borwein@newcastle.edu.au.

Pólya went on to reaffirm, nonetheless, that proof should certainly be taught in school.

We turn to observations, many of which have been fleshed out in coauthored books such as *Mathematics by Experiment* [10] and *Experimental Mathematics in Action* [3], in which we have noted the changing nature of mathematical knowledge and in consequence ask questions such as "How do we teach what and why to students?", "How do we come to believe and trust pieces of mathematics?", and "Why do we wish to prove things?" An answer to the last question is "That depends." Sometimes we wish insight and sometimes, especially with subsidiary results, we are more than happy with a certificate. The computer has significant capacities to assist with both.

Smail [27, p. 113] writes:

> the large human brain evolved over the past 1.7 million years to allow individuals to negotiate the growing complexities posed by human social living.

As a result, humans find various modes of argument more palatable than others and are more prone to make certain kinds of errors than others. Likewise, the well-known evolutionary psychologist Steve Pinker observes that language [24, p. 83] is founded on

> the ethereal notions of space, time, causation, possession, and goals that appear to make up a language of thought.

This remains so within mathematics. The computer offers scaffolding both to enhance mathematical reasoning, as with the recent computation connected to the Lie group E_8 (see http://www.aimath.org/E8/computerdetails.html), and to restrain mathematical error.

Experimental Methodology

Justice Potter Stewart's famous 1964 comment, "I know it when I see it," is the quote with which

The Computer as Crucible [13] starts. A bit less informally, by *experimental mathematics* we intend [10]:

(a) *gaining* insight and *intuition*;
(b) *visualizing* math principles;
(c) *discovering* new relationships;
(d) *testing* and especially *falsifying* conjectures;
(e) *exploring* a possible result to see if it *merits* formal proof;
(f) *suggesting* approaches for formal proof;
(g) *computing* replacing lengthy hand derivations;
(h) *confirming* analytically derived results.

Of these items, (a) through (e) play a central role, and (f) also plays a significant role for us but connotes computer-assisted or computer-directed proof and thus is quite distinct from *formal proof* as the topic of a special issue of the *Notices* in December 2008; see, e.g., [20].

Digital Integrity: I. For us, (g) has become ubiquitous, and we have found (h) to be particularly effective in ensuring the integrity of published mathematics. For example, we frequently check and correct identities in mathematical manuscripts by computing particular values on the LHS and RHS to high precision and comparing results—and then if necessary use software to repair defects.

As a first example, in a current study of "character sums" we wished to use the following result derived in [14]:

$$(1)\quad \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{(-1)^{m+n+1}}{(2m-1)(m+n-1)^3} \stackrel{?}{=} 4\,\mathrm{Li}_4\left(\frac{1}{2}\right)-\frac{51}{2880}\pi^4-\frac{1}{6}\pi^2\log^2(2)+\frac{1}{6}\log^4(2)+\frac{7}{2}\log(2)\zeta(3).$$

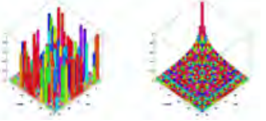
Here $\mathrm{Li}_4(1/2)$ is a polylogarithmic value. However, a subsequent computation to check results disclosed that, whereas the LHS evaluates to $-0.872929289\ldots$, the RHS evaluates to $2.509330815\ldots$. Puzzled, we computed the sum, as well as each of the terms on the RHS (sans their coefficients), to 500-digit precision, then applied the "PSLQ" algorithm, which searches for integer relations among a set of constants [16]. PSLQ quickly found the following:

$$(2)\quad \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{(-1)^{m+n+1}}{(2m-1)(m+n-1)^3} = 4\,\mathrm{Li}_4\left(\frac{1}{2}\right)-\frac{151}{2880}\pi^4-\frac{1}{6}\pi^2\log^2(2)+\frac{1}{6}\log^4(2)+\frac{7}{2}\log(2)\zeta(3).$$

In other words, in the process of transcribing (1) into the original manuscript, "151" had become "51". It is quite possible that this error would have gone undetected and uncorrected had we not been

Caption for attached graphic:

Mathematicians often work with matrices, which are arrays of numbers. When written on a page, a matrix can look like a sea of numbers, so any patterns that might occur in the numbers can be difficult to discern. More and more, mathematicians are turning to graphical representations of matrices, like the two examples here. By using color and form to indicate the values of the numbers in the matrix, these graphical representations can instantly give a sense of the patterns in the matrix. The first picture is a representation of a matrix in which the numbers exhibit a clear pattern; the second picture, by contrast, is a matrix in which the numbers are random. (Graphic by David Bailey and Jonathan Borwein. Request their permission before reproducing the graphic.)

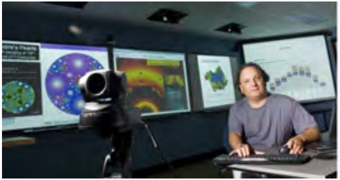


AMS Embargoed PR

CARMA's Mandate

Mathematics, as "the language of high technology" which underpins all facets of modern life and current Information and Communication Technology (ICT), is ubiquitous. No other research centre exists focussing on the implications of developments in ICT, present and future, for the practice of research mathematics.

- CARMA fills this gap through exploitation and development of techniques and tools for computer-assisted discovery and disciplined data-mining including mathematical visualization.



CARMA's Access Grid Room

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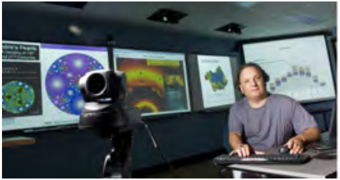

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CARMA's Objectives:

To perform R&D relating to the informed use of computers as an adjunct to mathematical discovery (including current advances in cognitive science, in information technology, operations research and theoretical computer science).

- of mathematics underlying computer-based decision support systems, particularly in automation and optimization of scheduling, planning and design activities, and to undertake mathematical modelling of such activities. (C-OPT, NUOR and partners)
- To promote and advise on use of appropriate tools (hardware, software, databases, learning object repositories, mathematical knowledge management, collaborative technology) in academia, education and industry.
- To make University of Newcastle a world-leading institution for Computer Assisted Research Mathematics and its Applications.[1]

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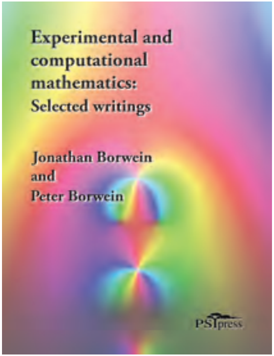
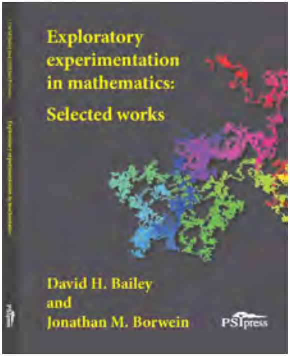
Communication and Computation: are entangled



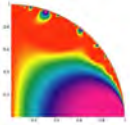


Communicating Mathematics (2008, 2010, 2012)

- 2012 *Science Communication* paper on AG seminars at `http://www.carma.newcastle.edu.au/jon/c2c11.pdf`

CARMA's Deep History

A co-evolution of symbolic/numeric (hybrid) computation, experimental maths, collaborative technology and **HPC**.



Experimentally-found modular fractal took 3 hrs to print

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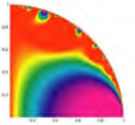

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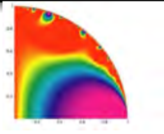

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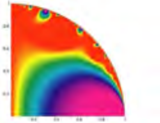

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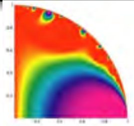

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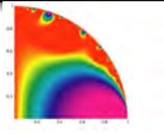

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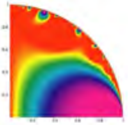

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2004-09 JMB opens D-Drive (Dalhousie Distributed Research Institute and Virtual Environment) with Canada Research Chair funding

2004 PBB opens IRMACS (www.irmacs.sfu.ca) with CFI funds

2008 CARMA funded/ opened as Univ. Priority Research Centre

2012 C-OPT founded. CARMA renewed to 2015? Then what?

[2] *J. Experimental Mathematics* founded in 1993.

CARMA's Structure

toc

Roughly **40** current Members and Associates:

- Steering Committee (Assoc Directors for Applied/Pure/OR)
- External Advisory Committee (IBM, Melbourne, LBNL)
- Members and Students from Newcastle
- Associate Members from Everywhere
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CARMA's AMSI AGR and **Inner Sanctum Rooms**

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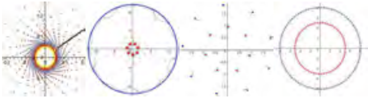
  - NUOR, SigmaOpt, Discrete Maths, Analysis and Number Theory



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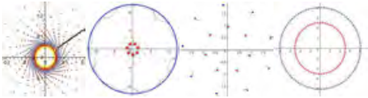

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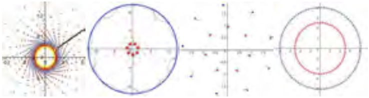

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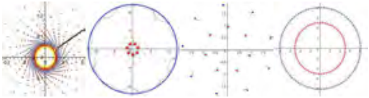

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Our Services Include

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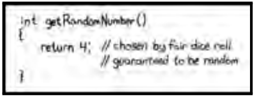
AGR Grid-enabled connected-rooms for classes, seminars, meetings:



V205 for dis-located collaboration;

V206 for co-located collaboration.

HPC 110 core MacPro Cluster and x-grid plus access to NSW and National computing services.

Web **Services** include:

- DocServer http://docserver.carma.newcastle.edu.au: CECM → DDRIVE → CARMA Archie → Mosaic → Google
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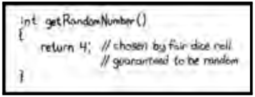

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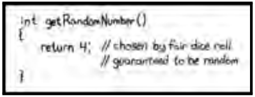

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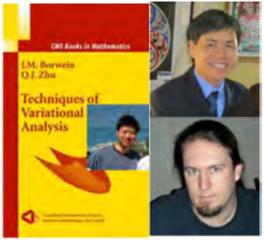

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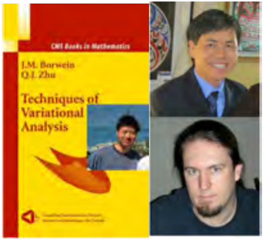

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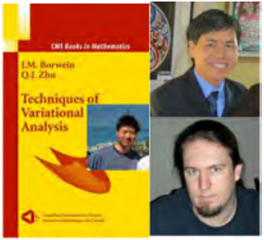

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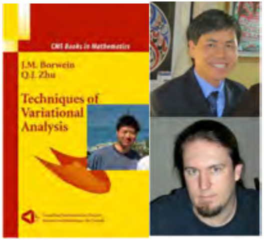

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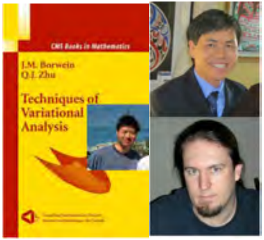

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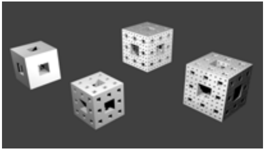
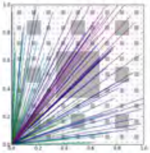
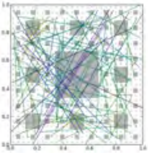
Symbolic-Numeric-Graphic Computation: SNAG



Square distance to origin (11/16) and between points (3/8) in fractal carpet





Michael Rose: work motivated by senile rat brains

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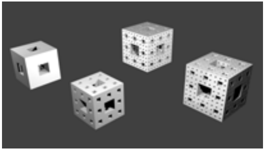

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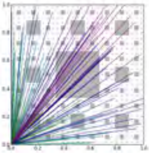

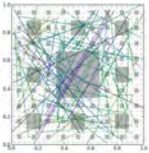

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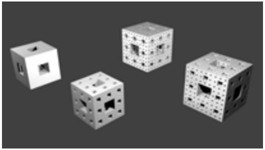

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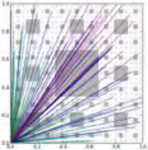

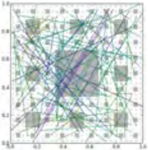

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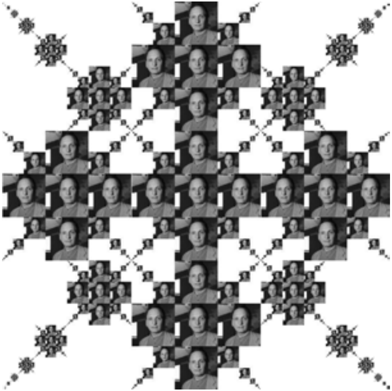
The Fractal Nature of Me: Examples of each of the 4 items follow



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2. 3 Optimization Texts: *Convex Functions* a 2011 Choice Outstanding Academic Title

3. Short Random Walks
4. Normality and Randomness: walking on numbers like π

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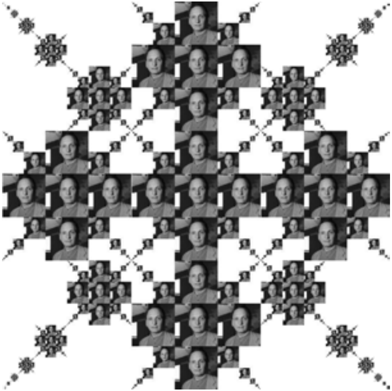

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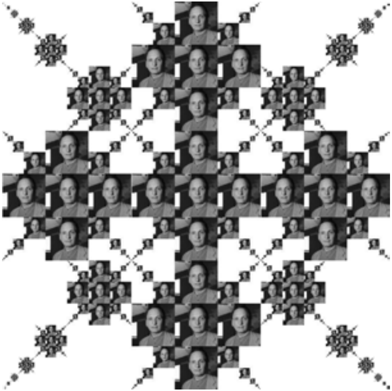

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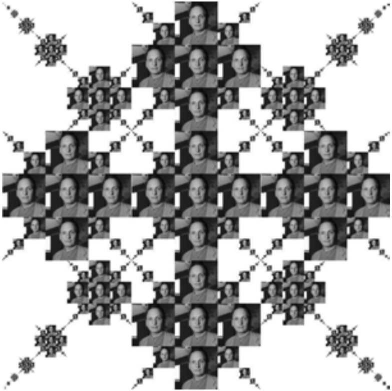

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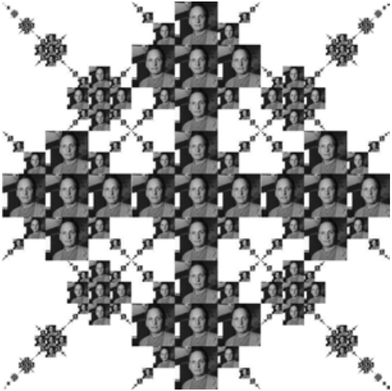

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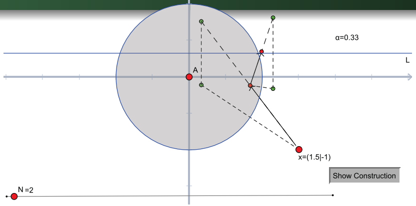

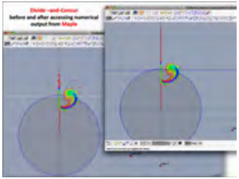

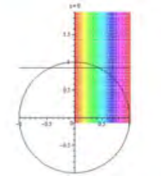

To find a point on a sphere and in an affine subspace

Briefly, a visual theorem is the graphical or visual output from a computer program — usually one of a family of such outputs — which the eye organizes into a coherent, identifiable whole and which is able to inspire mathematical questions of a traditional nature or which contributes in some way to our understanding or enrichment of some mathematical or real world situation.
— Davis, 1993, p. 333.

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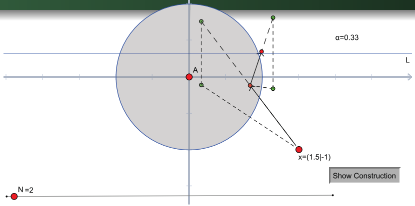

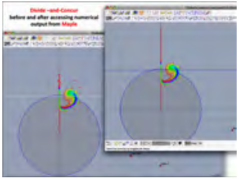

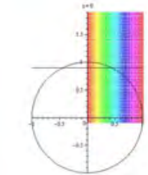

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3. Three Ramblers: A. Straub, J.J. Borwein, J. Wan

2011. AS won ACM-ISSAC Best Student Paper prize
JW was B.H. Neumann prize winner

3. Moments of Random Walks (Flights):

Definition (Moments and Challenging integrals)

For complex s the n-th moment function is

$$W_n(s) = \int_{[0,1]^n} \left| \sum_{k=1}^{n} e^{2\pi x_k i} \right|^s \mathrm{d}\boldsymbol{x}$$

$$= \int_{[0,1]^{n-1}} \left| 1 + \sum_{k=1}^{n-1} e^{2\pi x_k i} \right|^s \mathrm{d}(x_1, \ldots, x_{n-1})$$

Thus, $W_n := W_n(1)$ is the *expectation*.

- So

$$W_2 = 4 \int_0^{1/4} \cos(\pi x)\, \mathrm{d}x = \frac{4}{\pi}$$

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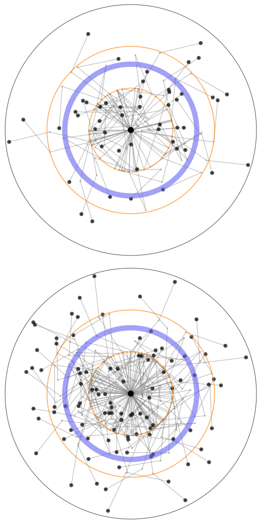
3. One 1500-step Walk in the plane: a familiar picture

2D and 3D lattice walks are different:

> *A drunk man will find his way home but a drunk bird may get lost forever.*
> — *Shizuo Kakutani*

3. 50, 100, 1000 3-step Walks: a less familiar picture?

toc SKIP



$$W_3(1) = \frac{16\sqrt[3]{4}\,\pi^2}{\Gamma(\frac{1}{3})^6} + \frac{3\Gamma(\frac{1}{3})^6}{8\sqrt[3]{4}\,\pi^4}$$

3. Moments of a Three Step Walk: in the complex plane

Theorem (Tractable hypergeometric form for W_3)

(a) *For $s \neq -3, -5, -7, \ldots$, we have*

$$W_3(s) = \frac{3^{s+3/2}}{2\pi}\,\beta\left(s+\frac{1}{2}, s+\frac{1}{2}\right)\,{}_3F_2\left(\left.{\frac{s+2}{2}, \frac{s+2}{2}, \frac{s+2}{2} \atop 1, \frac{s+3}{2}}\right|\frac{1}{4}\right). \tag{2}$$

(b) *For every natural number $k = 1, 2, \ldots,$*

$$W_3(-2k-1) = \frac{\sqrt{3}\,\binom{2k}{k}^2}{2^{4k+1}3^{2k}}\,{}_3F_2\left(\left.{\frac{1}{2}, \frac{1}{2}, \frac{1}{2} \atop k+1, k+1}\right|\frac{1}{4}\right).$$

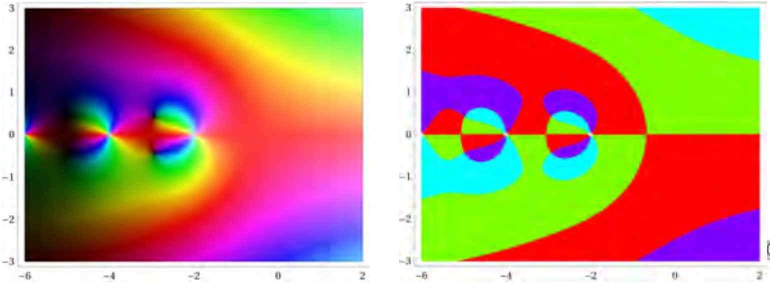
3. Moments of a Four Step Walk

Theorem (Meijer-G form for W_4)

For $\mathrm{Re}\ s > -2$ *and* s *not an odd integer*

$$W_4(s) = \frac{2^s}{\pi}\,\frac{\Gamma(1+\frac{s}{2})}{\Gamma(-\frac{s}{2})}\,G^{22}_{44}\left(\begin{matrix}1, \frac{1-s}{2}, 1, 1\\ \frac{1}{2}-\frac{s}{2}, -\frac{s}{2}, -\frac{s}{2}, -\frac{s}{2}\end{matrix}\,\middle|\,1\right). \qquad (3)$$

W_4 with phase colored continuously (L) and by quadrant (R)



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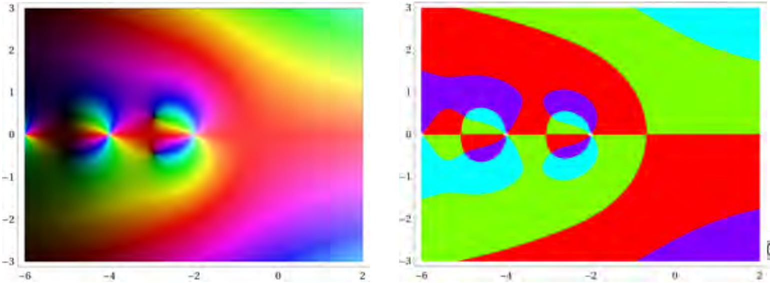
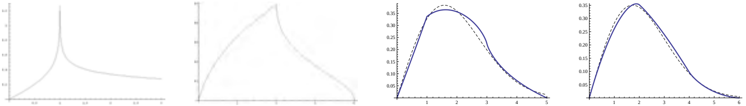


3. Density of a Three and Four Step Walk (BSW, 2010)

$$p_3(\alpha) = \frac{2\sqrt{3}\alpha}{\pi\,(3+\alpha^2)}\, {}_2F_1\left(\left.\begin{matrix}\frac{1}{3}, \frac{2}{3} \\ 1\end{matrix}\,\right|\, \frac{\alpha^2\left(9-\alpha^2\right)^2}{\left(3+\alpha^2\right)^3}\right)$$



For $n \geq 7$ the asymptotics $p_n(x) \sim \frac{2x}{n} e^{-x^2/n}$ are good.
(These are hard to draw.)

$$p_4(\alpha) = \frac{2}{\pi^2}\,\frac{\sqrt{16-\alpha^2}}{\alpha}\,\mathrm{Re}\ {}_3F_2\left(\left.\begin{matrix}\frac{1}{2}, \frac{1}{2}, \frac{1}{2} \\ \frac{5}{6}, \frac{7}{6}\end{matrix}\,\right|\, \frac{\left(16-\alpha^2\right)^3}{108\,\alpha^4}\right).$$

4. Pi Photo-shopped: a 2010 Pi Day Contest

Royal Society: "Nullius in Verba" (trust not in words)

Many mathematicians: "Noli Credere Pictis"

4. Life of Pi

- At the end of his story, Piscine (Pi) Molitor writes

Richard Parker (L) and Pi Molitor
Ang Lee's upcoming film
Life of Pi is now shooting
with a late 2012 3D-release

> I am a person who believes in form, in harmony of order. Where we can, we must give things a meaningful shape. For example — I wonder — could you tell my jumbled story in exactly one hundred chapters, not one more, not one less? I'll tell you, that's one thing I hate about my nickname, the way that number runs on forever. It's important in life to conclude things properly. Only then can you let go.

- We may not share the sentiment, but we should *celebrate* that Pi knows Pi to be irrational.

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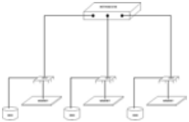
4. Why Pi? "Pi is Mount Everest."

What motivates modern computations of π — given that irrationality and transcendence of π were settled a century ago?

- One motivation is the raw challenge of harnessing the stupendous power of modern computer systems.



Programming is quite hard — especially on large, distributed memory computer systems: load balancing, communication needs, etc.

Substantial practical spin-offs accrue:

- Accelerating computations of π sped up the fast Fourier transform (FFT) — heavily used in science and engineering.
- Also to bench-marking and proofing computers, since brittle algorithms make better tests.

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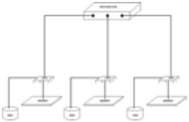

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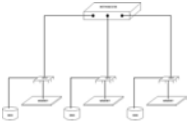

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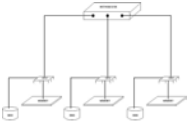

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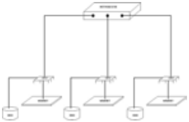

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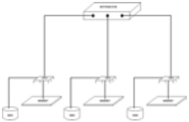

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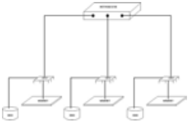

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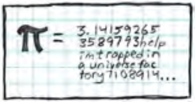
John von Neumann so prompted **ENIAC** computation of π and e — and e showed anomalies.



- Kanada, e.g., made detailed statistical analysis — without success — hoping some test suggests π is **not** normal.
  - The 10 decimal digits ending in position one trillion are **6680122702**, while the 10 hexadecimal digits ending in position one trillion are **3F89341CD5**.
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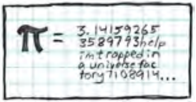

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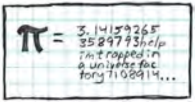

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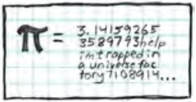

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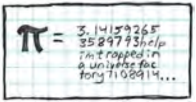

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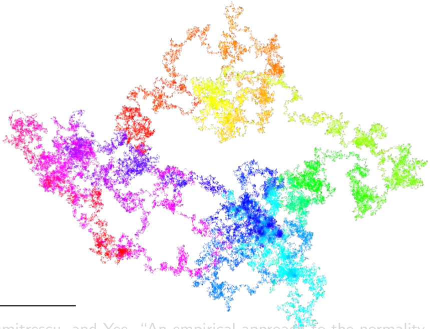
Fran Aragon's 2.873 GB walk on a 200 billion binary digits of Pi

- A $372,224 \times 290,218$ pixel image at http://gigapan.com/gigapans/106803/
- A Poisson inter-arrival time model applied to 15.925 trillion bits gives: probability Pi is not normal less than one part in is 10^{3600} .[3]

At work Haifa, May 2012



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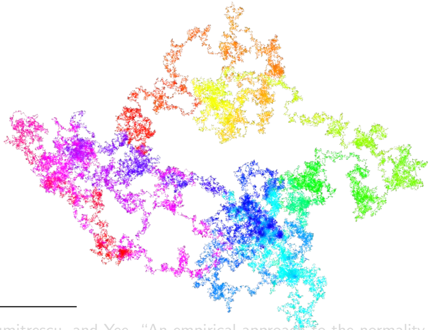

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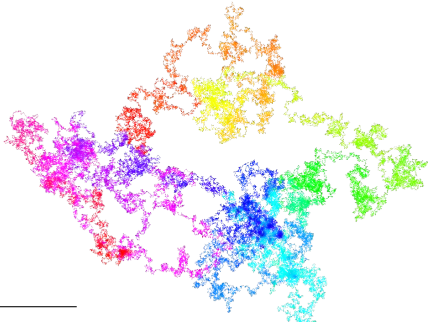

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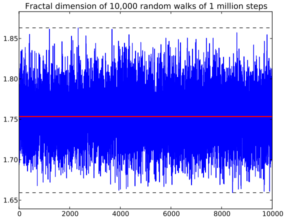
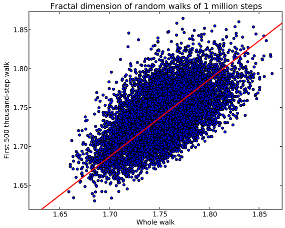
4. Pi seems Random: Some million step bit walks

toc
SKIP

Euler's constant and a pseudo-random number





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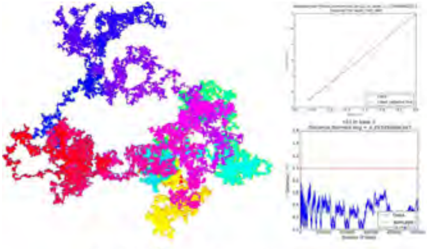
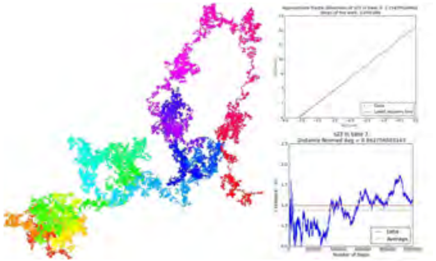
- b-**Normal**: all length n b-ary strings occur with prob. $1/b^n$
- In base 2 Stoneham's number is provably normal (Left)
- It may be normal base 3 (Right)





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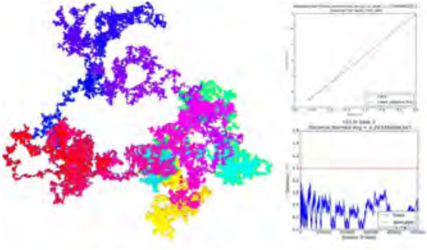

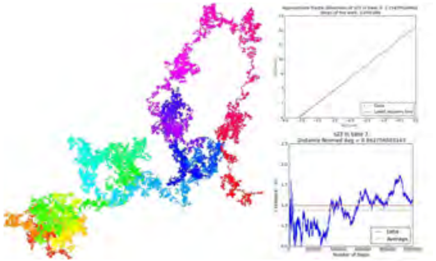

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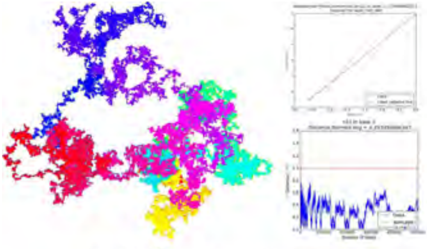

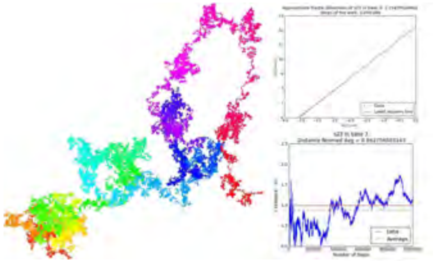

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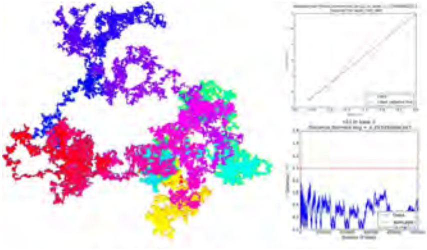
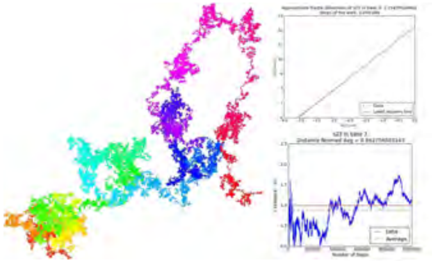
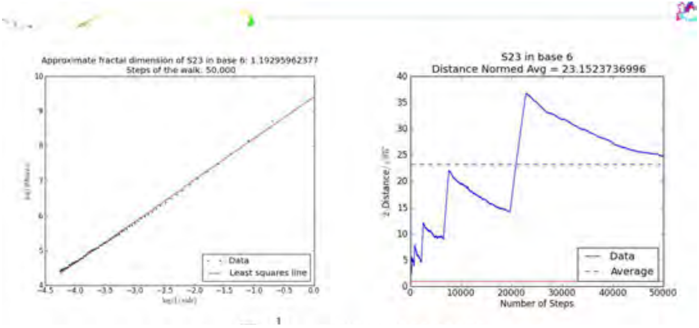




4. Pi Seems Normal: Comparisons to Stoneham's number, II

- Stoneham's number is provably abnormal base 6 (there are way too many zeros).
- And in many other bases. We should have drawn pictures earlier!



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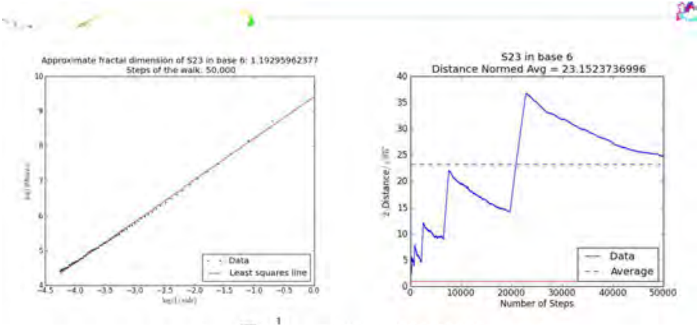

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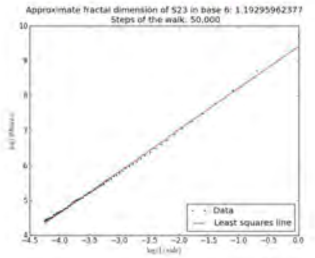
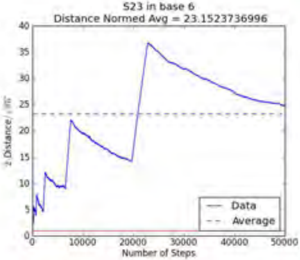
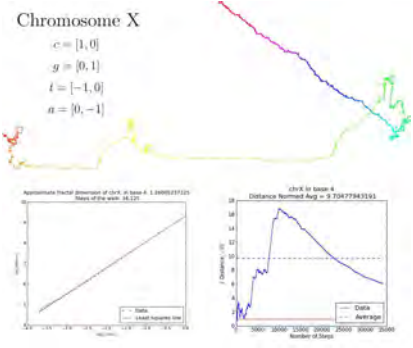
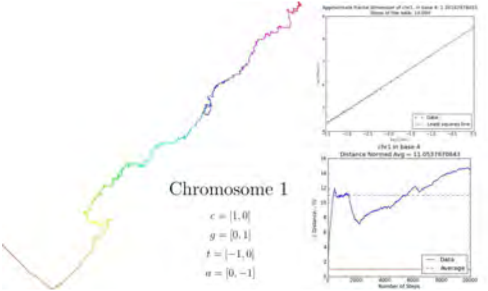




4. Pi seems Random and Normal: Compared to Human Genomes

Genomes are 'just' base four numbers.

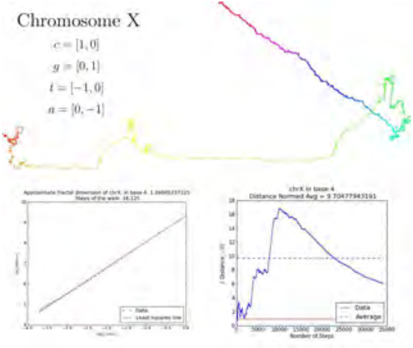


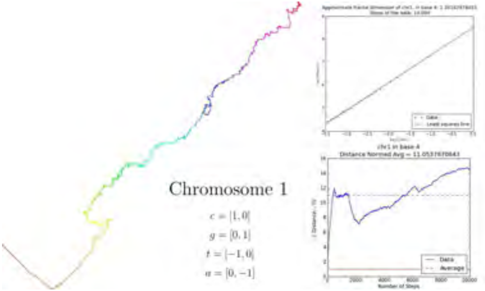


The X Chromosome (34K) and Chromosome One (10K).

4. Pi seems Random and Normal: Compared to Human Genomes

Genomes are 'just' base four numbers.





The X Chromosome (34K) and Chromosome One (10K).

4. Pi Seems Normal: Comparisons to other provably normal numbers


Erdös-Copeland number (concatenated primes, base 2) and
Champernowne number (concatenated integers, base 4).

- All pictures thanks to Fran Aragon and Jake Fountain http://www.carma.newcastle.edu.au/jon/numtools.pdf

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4. Pi is Still Mysterious: Things we don't know about Pi

We do not 'know' (in the sense of being able to prove) whether ....

- The simple continued fraction for Pi is unbounded.
  - Euler found the **292**.
- There are infinitely many sevens in the decimal expansion of Pi.
- There are infinitely many ones in the ternary expansion of Pi.
- There are equally many zeroes and ones in the binary expansion of Pi.
- Or pretty much anything I have not told you.

$$\pi = 3 + \cfrac{1}{7 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{292 + \cfrac{1}{1 + \cfrac{1}{1 + \ldots}}}}}}$$

$$e = 2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{6 + \cfrac{1}{1 + \ldots}}}}}}}}}$$

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4. Animation, Simulation and Stereo . . .

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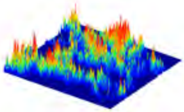

Cinderella, 3.14 min of Pi, Catalan's constant and Passive Three D

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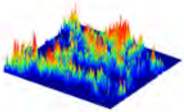

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Thank You to All

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3. Pi Day 2012: http://carma.newcastle.edu.au/jon/piday.pdf
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2010: Communication is not yet always perfect

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