Seeing Things by Walking on Real Numbers

Jonathan Borwein FRSC FAAS FAA FBAS
(Joint work with Francisco Aragón, David Bailey and Peter Borwein)

CARMA

THE UNIVERSITY OF NEWCASTLE AUSTRALIA

School of Mathematical & Physical Sciences
The University of Newcastle, Australia

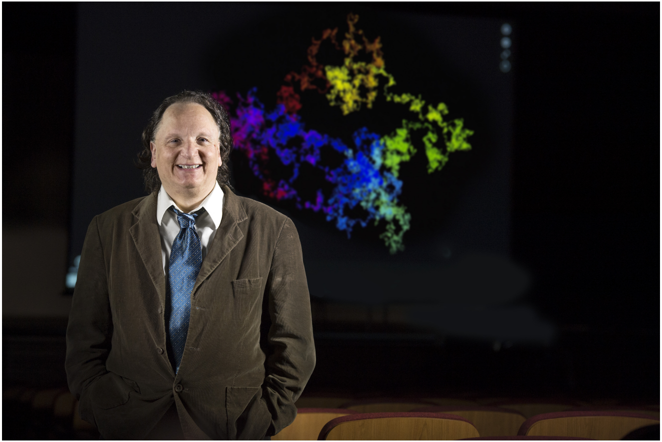
http://carma.newcastle.edu.au/meetings/evims/

For 2014 Presentations
Revised 10-04-2014

Contents:

One message is "Try drawing numbers"

Me and my collaborators

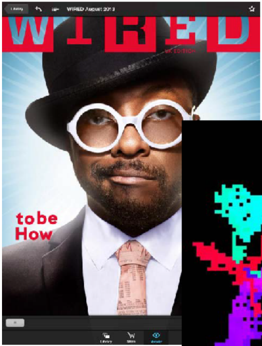


MAA 3.14

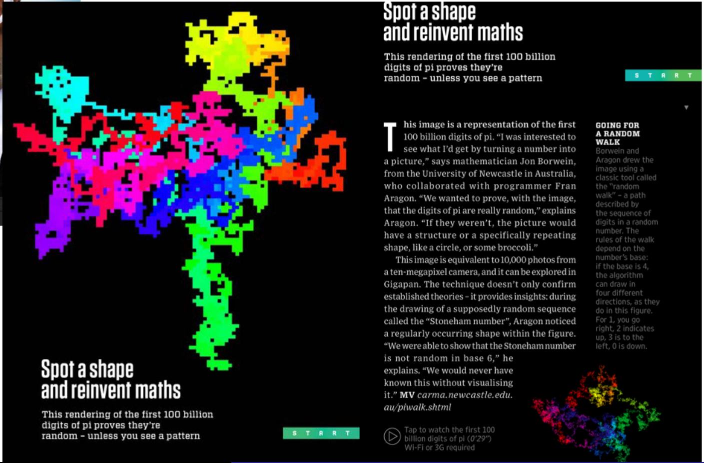
http://www.carma.newcastle.edu.au/jon/pi-monthly.pdf

My collaborators

Outreach: images and animations led to high-level research which went viral



Wired UK August 2013



Spot a shape and reinvent maths

This rendering of the first 100 billion digits of pi proves they're random – unless you see a pattern

START

This image is a representation of the first 100 billion digits of pi. "I was interested to see what I'd get by turning a number into a picture," says mathematician Jon Borwein, from the University of Newcastle in Australia, who collaborated with programmer Fran Aragon. "We wanted to prove, with the image, that the digits of pi are really random," explains Aragon. "If they weren't, the picture would have a structure or a specifically repeating shape, like a circle, or some broccoli."

This image is equivalent to 10,000 photos from a ten-megapixel camera, and it can be explored in Gigapan. The technique doesn't only confirm established theories – it provides insights: during the drawing of a supposedly random sequence called the "Stoneham number", Aragon noticed a regularly occurring shape within the figure. "We were able to show that the Stoneham number is not random in base 6," he explains. "We would never have known this without visualising it." **MV** *carma.newcastle.edu.au/piwalk.shtml*

GOING FOR A RANDOM WALK
Borwein and Aragon drew the image using a classic tool called the "random walk" – a path described by the sequence of digits in a random number. The rules of the walk depend on the number's base: if the base is 4, the algorithm can draw in four different directions, as they do in this figure. For 1, you go right, 2 indicates up, 3 is to the left, 0 is down.

Tap to watch the first 100 billion digits of pi (0'29")
Wi-Fi or 3G required

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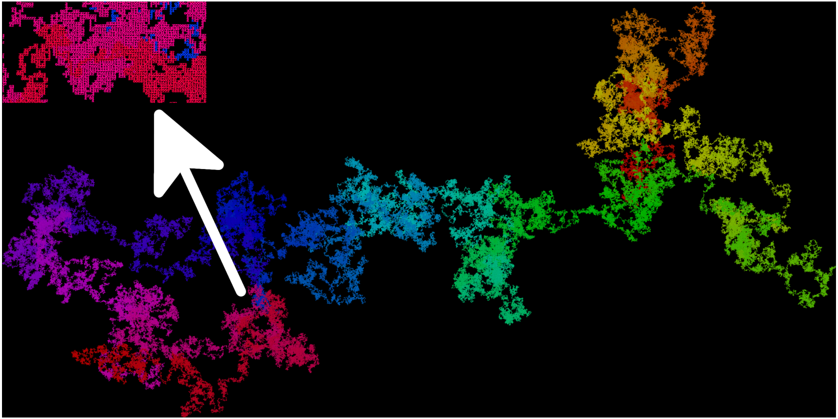
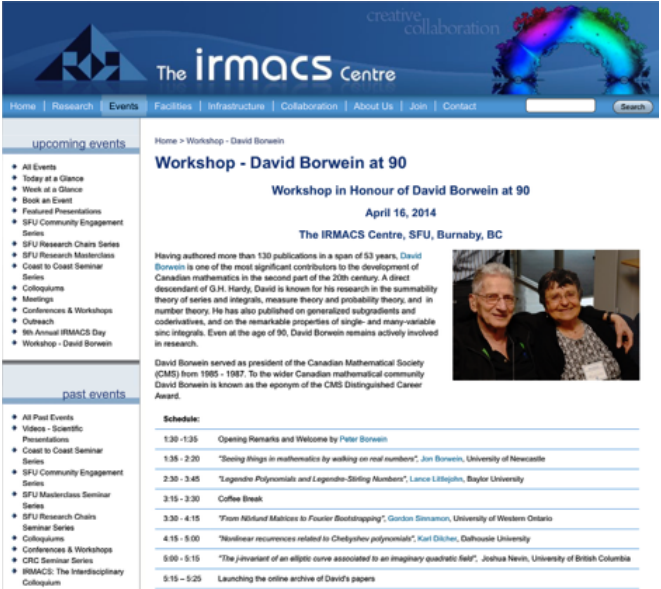


- 100 billion base four digits of π on Gigapan
- Really big pictures are often better than movies

Contents

Dedication: To my father and colleague David Borwein (1924 –)



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Workshop - David Borwein at 90

Workshop in Honour of David Borwein at 90

April 16, 2014

The IRMACS Centre, SFU, Burnaby, BC

Having authored more than 130 publications in a span of 53 years, David Borwein is one of the most significant contributors to the development of Canadian mathematics in the second part of the 20th century. A direct descendant of G.H. Hardy, David is known for his research in the summability theory of series and integrals, measure theory and probability theory, and in number theory. He has also published on generalized subgradients and coderivatives, and on the remarkable properties of single- and many-variable sinc integrals. Even at the age of 90, David Borwein remains actively involved in research.

David Borwein served as president of the Canadian Mathematical Society (CMS) from 1985 - 1987. To the wider Canadian mathematical community David Borwein is known as the eponym of the CMS Distinguished Career Award.

Schedule:

1:30 -1:35	Opening Remarks and Welcome by Peter Borwein
1:35 - 2:20	*"Seeing things in mathematics by walking on real numbers"*, Jon Borwein, University of Newcastle
2:30 - 3:45	*"Legendre Polynomials and Legendre-Stirling Numbers"*, Lance Littlejohn, Baylor University
3:15 - 3:30	Coffee Break
3:30 - 4:15	*"From Nörlund Matrices to Fourier Bootstrapping"*, Gordon Sinnamon, University of Western Ontario
4:15 - 5:00	*"Nonlinear recurrences related to Chebyshev polynomials"*, Karl Dilcher, Dalhousie University
5:00 - 5:15	*"The j-invariant of an elliptic curve associated to an imaginary quadratic field"*, Joshua Nevin, University of British Columbia
5:15 – 5:25	Launching the online archive of David's papers

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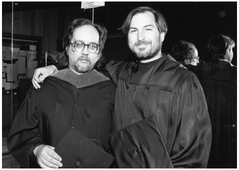


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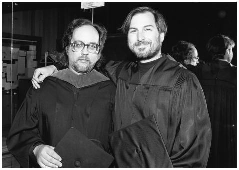

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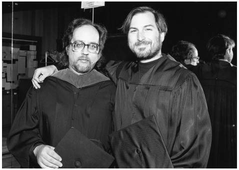


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  - and the *iPod shuffle*

http://en.wikipedia.org/wiki/Richard_Crandall

Some early conclusions:

So I am sure they get made

Key ideas: randomness, normality of numbers, planar walks, and fractals

How not to experiment

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- using computer algebra, numerical computation and graphics: SNaG
- computations, tables and pictures are experimental data
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DHB and JMB, **Exploratory Experimentation in Mathematics** (2011), www.ams.org/notices/201110/rtx111001410p.pdf

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I imagine the world conqueror must feel thus, who, after one kingdom is scarcely conquered, stretches out his arms for others.

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(1777-1855)

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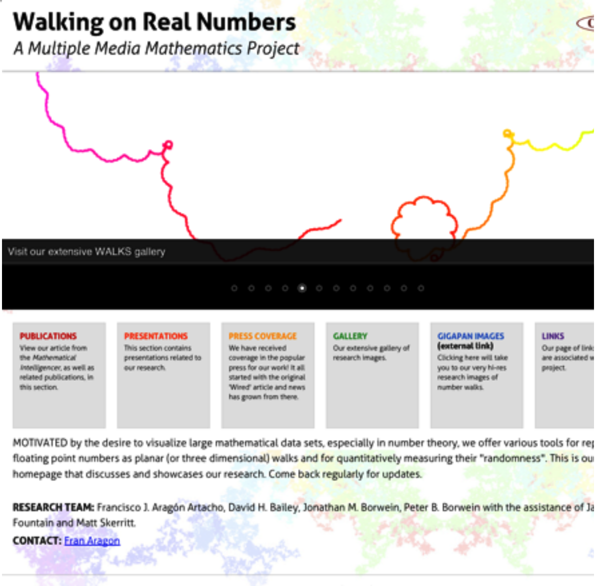
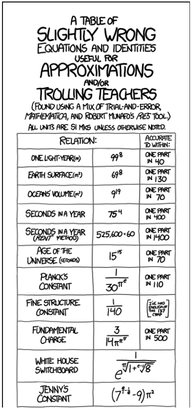
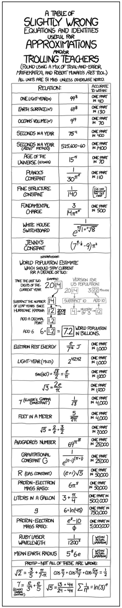
- In an **1808** letter to his friend Farkas (father of Janos Bolyai)
- Archimedes, Euler, Gauss are the big three







Almost all I mention is at `http://carma.newcastle.edu.au/walks/`

A surprising fan?

26-07-2013

He [David Attenborough] described current pop music as "hugely sexual and even lets slip that if he were not one of the world's most famous broadcasters, he would like to try his hand at academia. "I wish I was a mathematician, he said. "I know a mathematician would talk about the beauty of an equation. And you can sense that when you hear a five-part fugue by Bach, which also has a mathematical beauty.

www.independent.co.uk/arts-entertainment/tv/features/

when-bjrk-met-attenborough-the-icelandic-punk-the-national-treasure-and-a-display-of-rather-remarkable-human-

html

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We shall explore things like: How random is Pi?

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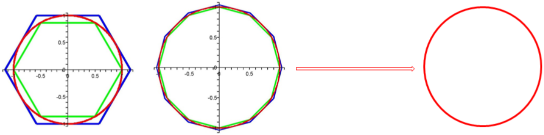

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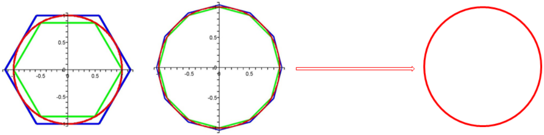

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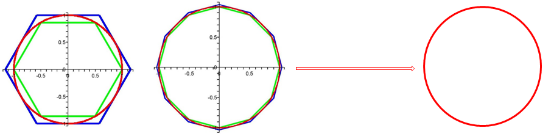
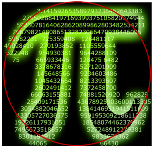
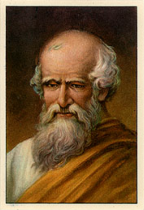


$\mathbf{6} \mapsto \mathbf{12} \mapsto 24 \mapsto 48 \mapsto \mathbf{96}$ to obtain the estimate

$$3\frac{10}{71} < \pi < 3\frac{10}{70}.$$



Where Greece was: Magna Graecia



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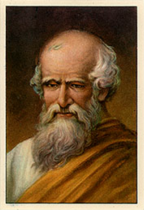

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6. Ephesus
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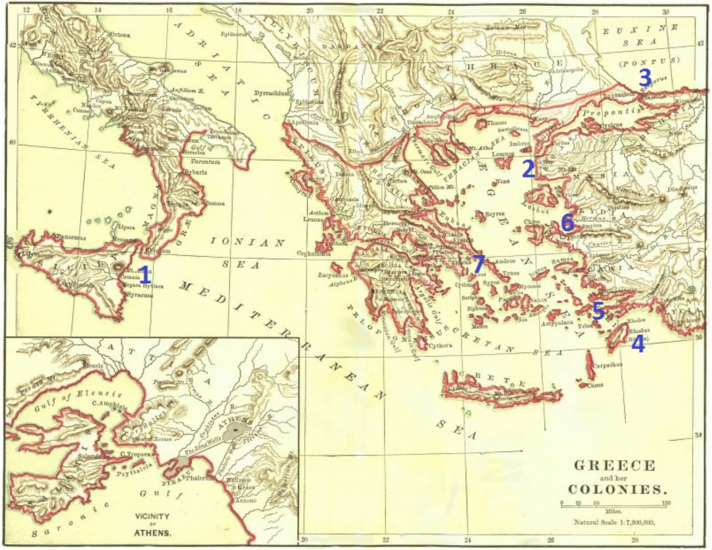

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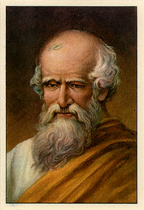

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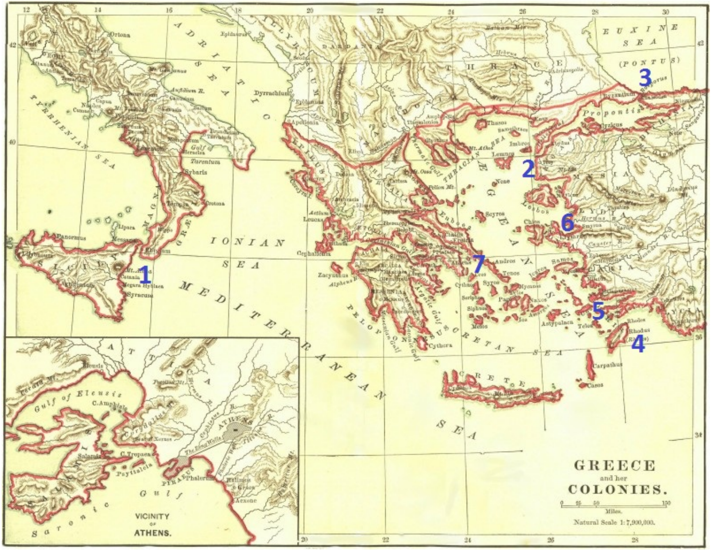

The others of the Seven Wonders of the Ancient World: Lighthouse of Alexandria, Pyramids of Giza, Gardens of Babylon

Randomness

- The digits expansions of $\pi, e, \sqrt{2}$ appear to be "random":

$$\pi = 3.14159265358979323846264338327950288419716939937 5\ldots$$

$$e = 2.71828182845904523536028747135266249775724709369 9\ldots$$

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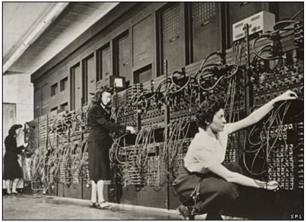
- **1949 ENIAC** (*Electronic Numerical Integrator and Calculator*) computed of π to **2,037** decimals (in **70** hours)—proposed by polymath John von Neumann (**1903-1957**) to shed light on distribution of π (and of e).



Two continued fractions

Change representations often

Gauss map. Remove the integer, invert the fraction and repeat: for 3.1415926 and 2.7182818 to get the fractions below.

$$\pi = 3 + \cfrac{1}{7 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{292 + \cfrac{1}{1 + \cfrac{1}{1 + \ldots}}}}}}$$

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$$e = 2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{6 + \cfrac{1}{1 + \ldots}}}}}}}}}$$

Leonhard Euler (**1707-1783**) named e and π.

"Lisez Euler, lisez Euler, c'est notre maître à tous." Simon Laplace (**1749-1827**)

Are the digits of π random?

Digit	Ocurrences
0	99,993,942
1	99,997,334
2	100,002,410
3	99,986,911
4	**100,011**,958
5	**99,998**,885
6	100,010,387
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Total	**1,000,000,000**

Table : Counts of first billion digits of π. Second half is 'right' for law of large numbers.

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- There are infinitely many ones in the ternary expansion of π

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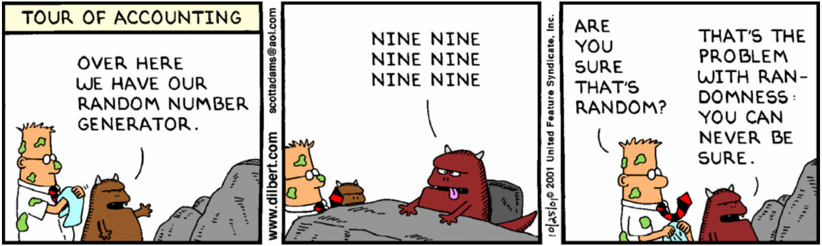
- There are infinitely many sevens in the decimal expansion of π
- There are infinitely many ones in the ternary expansion of π
- There are equally many zeroes and ones in the binary expansion of π
- Or pretty much anything else...

What is “random”? A hard question



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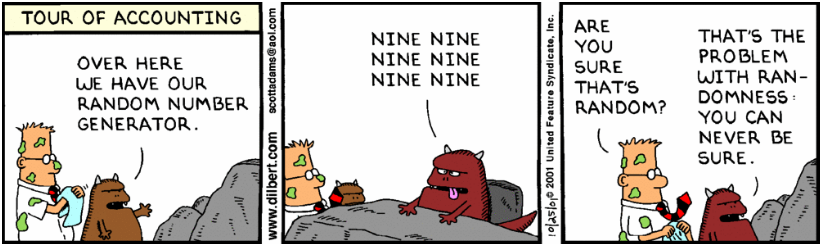

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- Unpredictable (fair dice or coin-flips)?
- Without structure (noise)?
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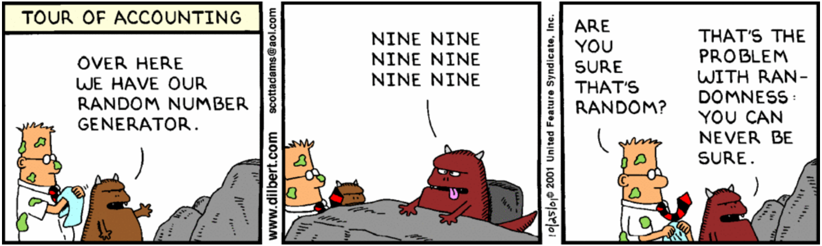

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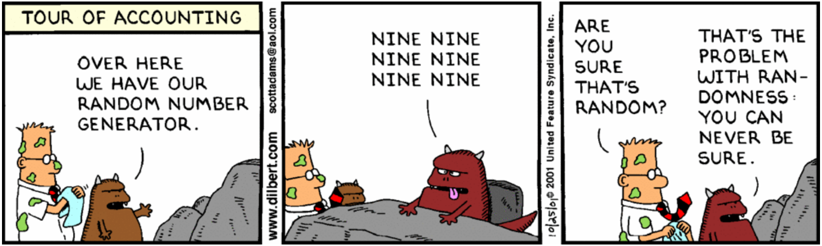

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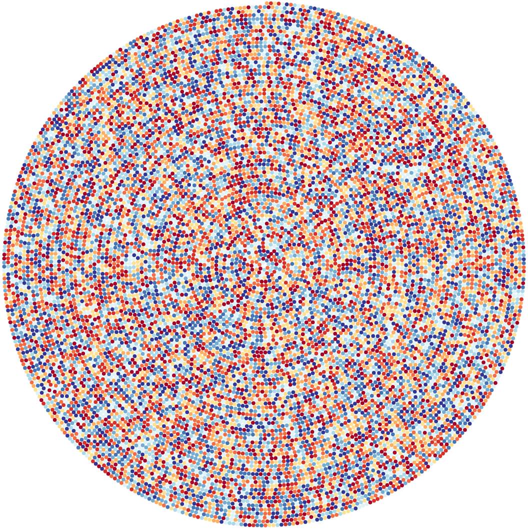
Best Theorem [BBCP, 04] (Feeble but hard) Asymptotically all degree d algebraics have at least $n^{1/d}$ ones in binary (should be $n/2$)

Randomness in Pi?

http://mkweb.bcgsc.ca/pi/art/



Normality

A property random numbers must possess

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A real constant α is b-normal if, given the positive integer $b \geq 2$ (the base), every m-long string of base-b digits appears in the base-b expansion of α with precisely the expected limiting frequency $1/b^m$.

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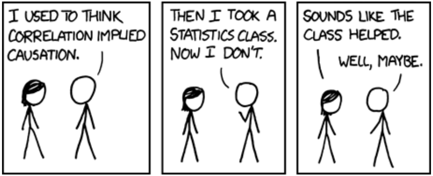

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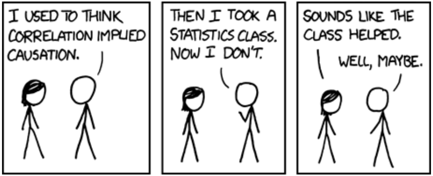
- Indeed, almost all real numbers are b-normal simultaneously for all positive integer bases (“absolute normality”).



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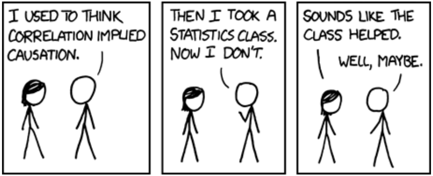
- Indeed, almost all real numbers are b-normal simultaneously for all positive integer bases (“absolute normality”).
- Unfortunately, it has been very difficult to prove normality for any number in a given base b, much less all bases simultaneously.



Normal numbers *concatenation* numbers

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- Normality proofs are not known for $\pi, e, \log 2, \sqrt{2}$ etc.

Contents

Is π 10-normal?

String	Occurrences	String	Occurrences	String	Occurrences
0	99,993,942	00	10,004,524	000	1,000,897
1	99,997,334	01	9,998,250	001	1,000,758
2	100,002,410	02	9,999,222	002	1,000,447
3	99,986,911	03	10,000,290	003	1,001,566
4	100,011,958	04	10,000,613	004	1,000,741
5	**99,998,885**	05	**10,002,048**	005	**1,002,881**
6	100,010,387	06	9,995,451	006	999,294
7	99,996,061	07	9,993,703	007	998,919
8	100,001,839	08	10,000,565	008	999,962
9	100,000,273	09	9,999,276	009	999,059
		10	9,997,289	010	998,884
		11	9,997,964	011	1,001,188
		⋮	⋮	⋮	⋮
		99	10,003,709	099	999,201
				⋮	⋮
				999	1,000,905
TOTAL	1,000,000,000	**TOTAL**	1,000,000,000	**TOTAL**	1,000,000,000

Table : Counts for the first billion digits of π.

Is π 16-normal

That is, in Hex?

$\hookleftarrow$ Counts of first trillion hex digits

0	62499881108
1	62500212206
2	62499924780
3	62500188844
4	62499807368
5	62500007205
6	62499925426
7	62499878794
8	**62500**216752
9	62500120671
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- They are $353CB3F7F0C9ACCFA9AA215F2$

See www.karrels.org/pi/index.html

Modern π Calculation Records: and IBM Blue Gene/L at LBL

Name	Year	Correct Digits
Miyoshi and Kanada	1981	2,000,036
Kanada-Yoshino-Tamura	1982	16,777,206
Gosper	1985	17,526,200
Bailey	Jan. 1986	29,360,111
Kanada and Tamura	Sep. 1986	33,554,414
Kanada and Tamura	Oct. 1986	67,108,839
Kanada et. al	Jan. 1987	134,217,700
Kanada and Tamura	Jan. 1988	201,326,551
Chudnovskys	May 1989	480,000,000
Kanada and Tamura	Jul. 1989	536,870,898
Kanada and Tamura	Nov. 1989	1,073,741,799
Chudnovskys	Aug. 1991	2,260,000,000
Chudnovskys	May 1994	4,044,000,000
Kanada and Takahashi	Oct. 1995	6,442,450,938
Kanada and Takahashi	Jul. 1997	51,539,600,000
Kanada and Takahashi	Sep. 1999	206,158,430,000
Kanada-Ushiro-Kuroda	Dec. 2002	1,241,100,000,000
Takahashi	Jan. 2009	1,649,000,000,000
Takahashi	April 2009	2,576,980,377,524
Bellard	Dec. 2009	2,699,999,990,000
Kondo and Yee	Aug. 2010	**5,000,000,000,000**
Kondo and Yee	Oct. 2011	**10,000,000,000,000**
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What BBP Is? Reverse Engineered Mathematics

This is based on the following then new formula for π:

$$\pi = \sum_{i=0}^{\infty} \frac{1}{16^i} \left(\frac{4}{8i+1} - \frac{2}{8i+4} - \frac{1}{8i+5} - \frac{1}{8i+6} \right) \tag{1}$$

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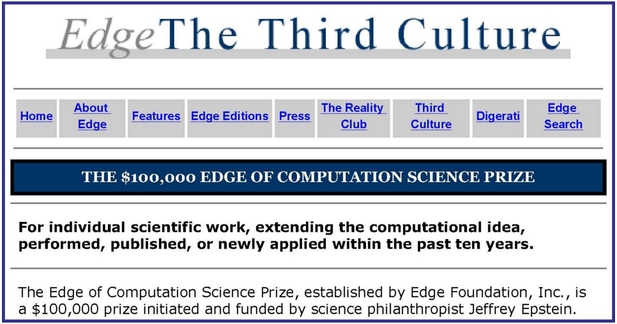
$$\pi = 4\,{}_2F_1\left(1, \frac{1}{4}; \frac{5}{4}, -\frac{1}{4}\right) + 2\tan^{-1}\left(\frac{1}{2}\right) - \log 5$$

where ${}_2F_1(1, 1/4; 5/4, -1/4) = 0.955933837\ldots$ is a Gaussian hypergeometric function.

Edge of Computation Prize Finalist



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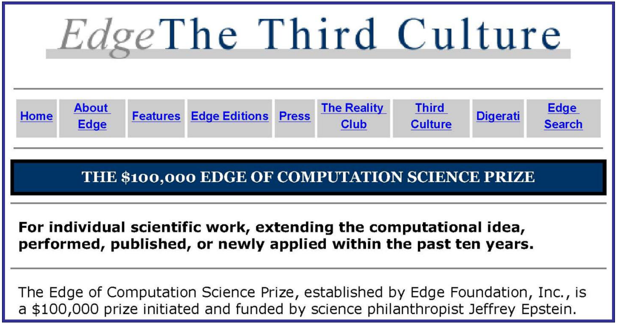

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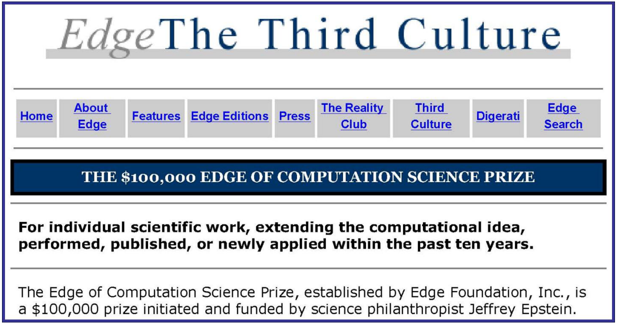

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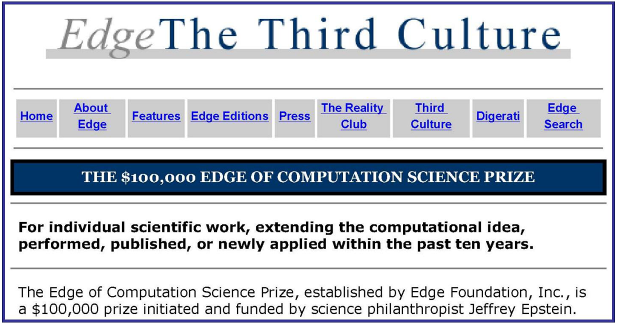

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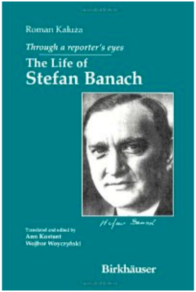
- Won by David Deutsch — discoverer of Quantum Computing.

Stefan Banach (1892-1945) Another Nazi casuality

A mathematician is a person who can find analogies between theorems; a better mathematician is one who can see analogies between proofs and the best mathematician can notice analogies between theories. [1]

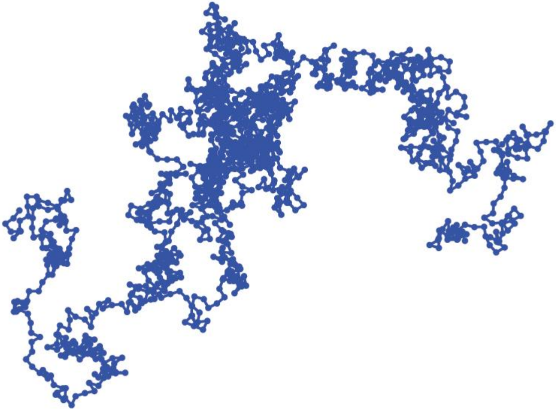


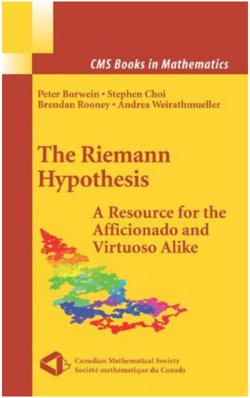
[1] Only the best get stamps. Quoted in `www-history.mcs.st-andrews.ac.uk/Quotations/Banach.html`.

Contents

One 1500-step ramble: a familiar picture

Liouville function





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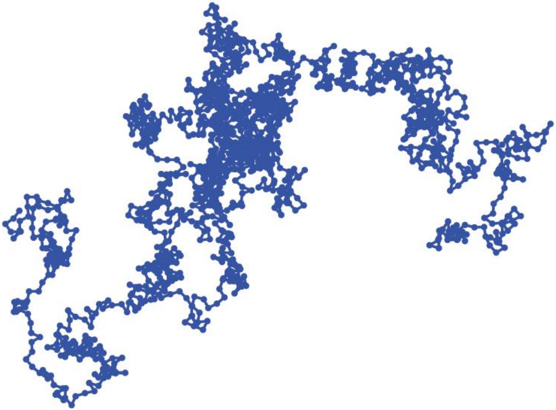

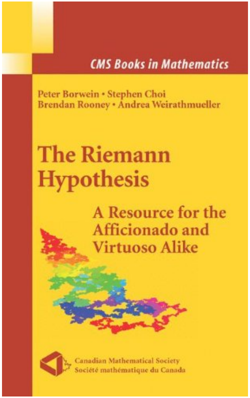

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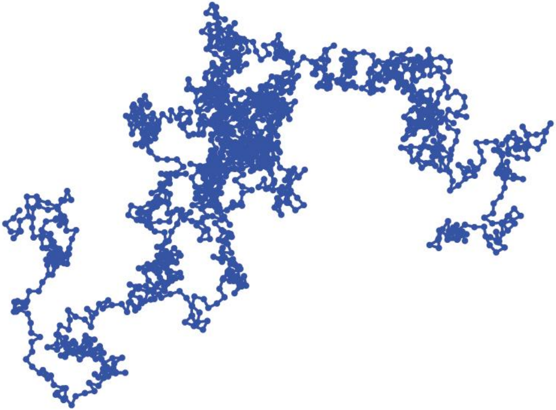

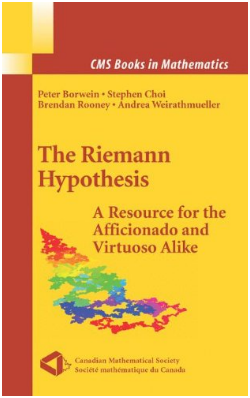

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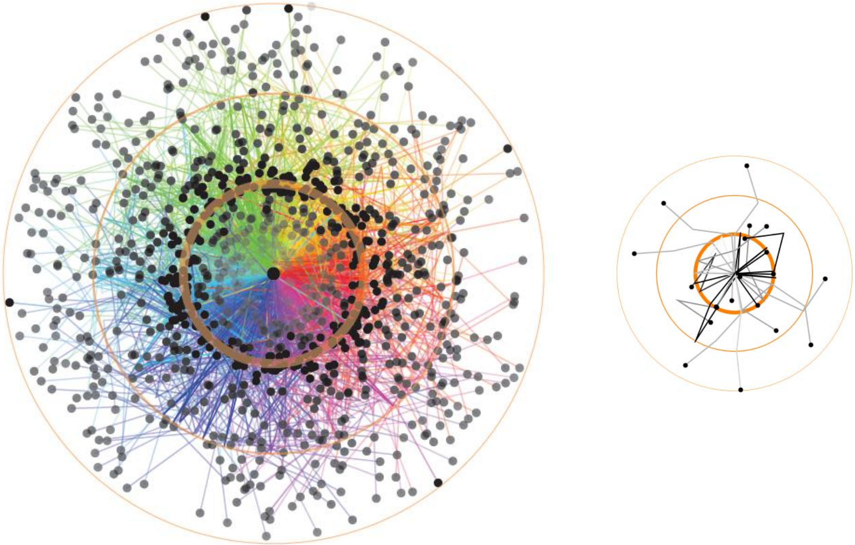
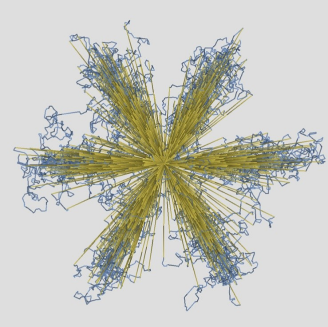
- 1D or 2D *lattice*: probability one of returning to the origin.

1000 three-step rambles: a less familiar picture?



Art meets science

AAAS & Bridges conference



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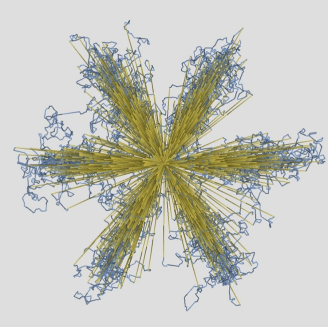
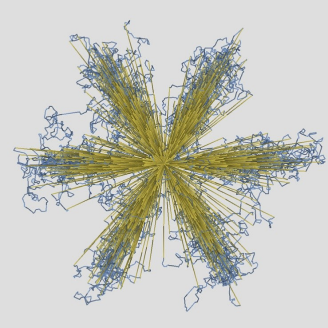


A visualization of six routes that 1000 ants took after leaving their nest in search of food. The jagged blue lines represent the breaking off of random ants in search of seeds.

(Nadia Whitehead 2014-03-25 16:15)

Art meets science

AAAS & Bridges conference

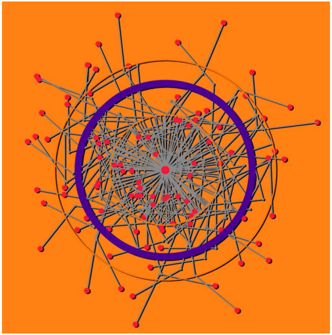


A visualization of six routes that 1000 ants took after leaving their nest in search of food. The jagged blue lines represent the breaking off of random ants in search of seeds.

(Nadia Whitehead 2014-03-25 16:15)

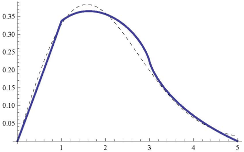
(JonFest 2011 Logo) *Three-step random walks. The (purple) expected distance travelled is 1.57459 ...*

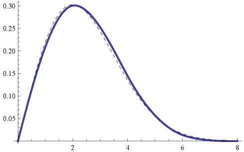
The closed form W_3 *is given below.*



$$W_3 = \frac{16\sqrt[3]{4}\,\pi^2}{\Gamma(\frac{1}{3})^6} + \frac{3\Gamma(\frac{1}{3})^6}{8\sqrt[3]{4}\,\pi^4}$$

A Little History: From a vast literature





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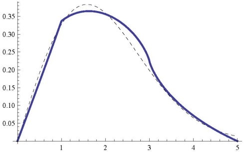

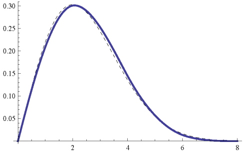

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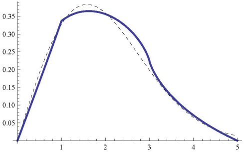

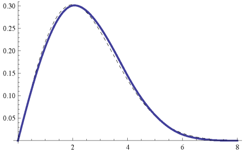

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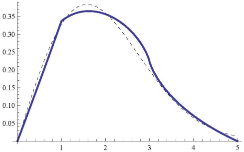

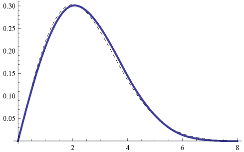

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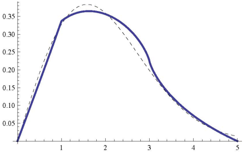

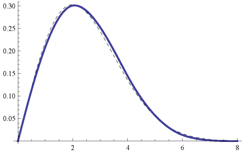

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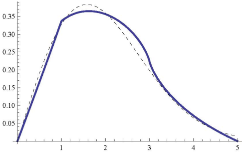

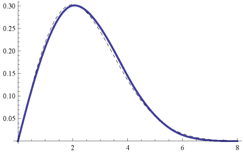

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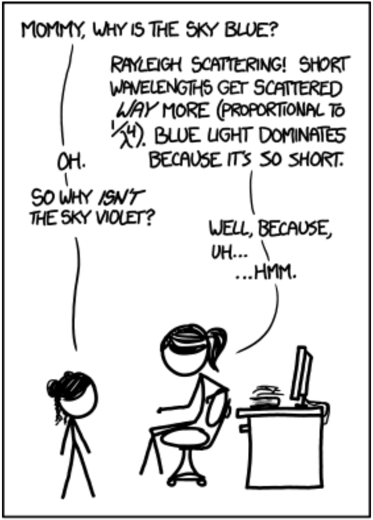
- appear in graph theory, quantum chemistry, in quantum physics as hexagonal and diamond lattice integers, etc ...

Why is the sky blue?



MY HOBBY: TEACHING TRICKY QUESTIONS TO THE CHILDREN OF MY SCIENTIST FRIENDS.

Contents

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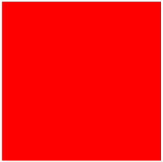
Pick a random number in $\{0, 1, 2, 3\}$ and move according to $0 = \rightarrow$, $1 = \uparrow$, $2 = \leftarrow$, $3 = \downarrow$



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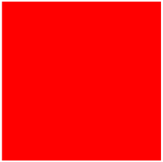

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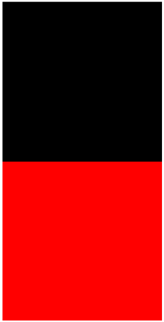

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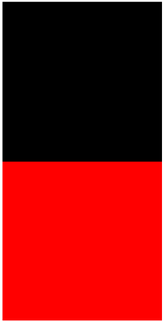

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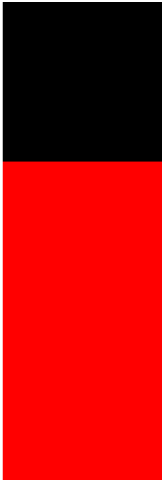

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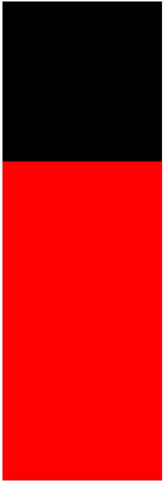

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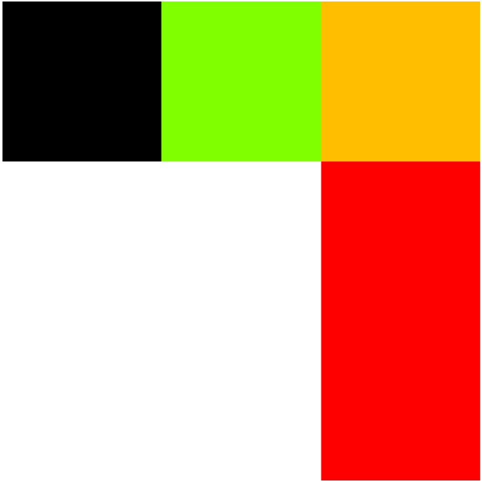

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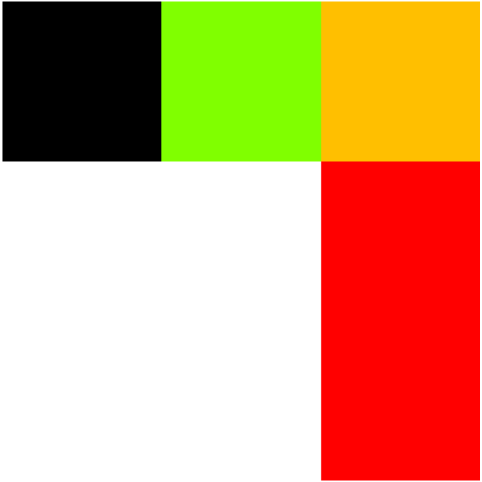

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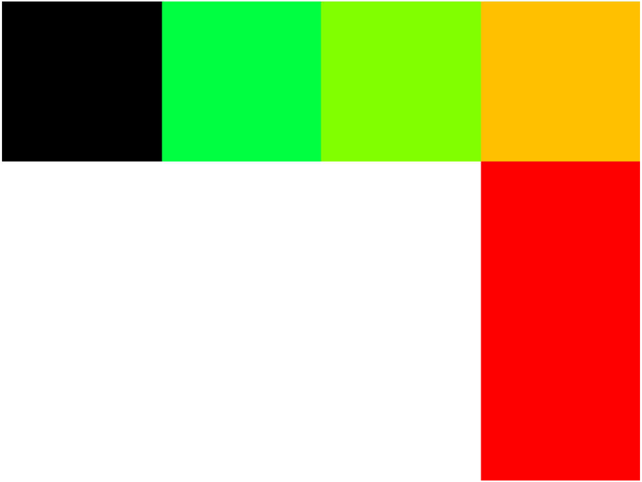

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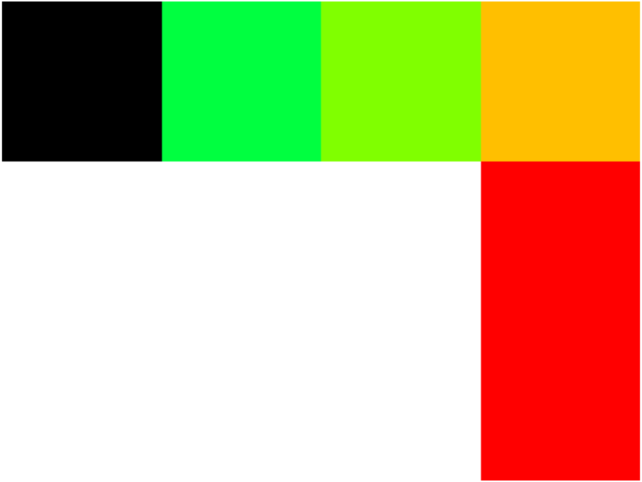

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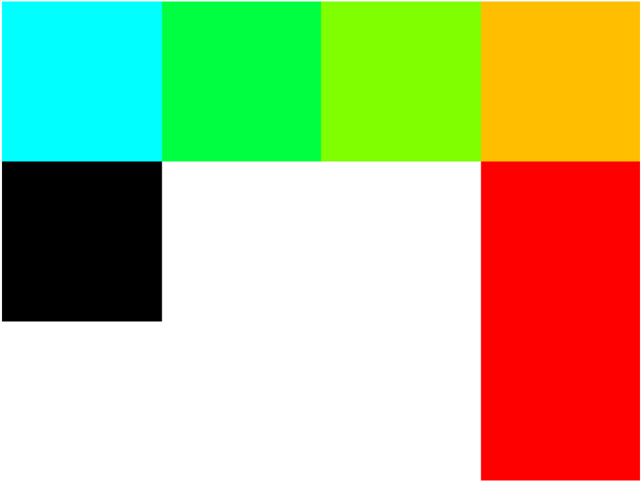

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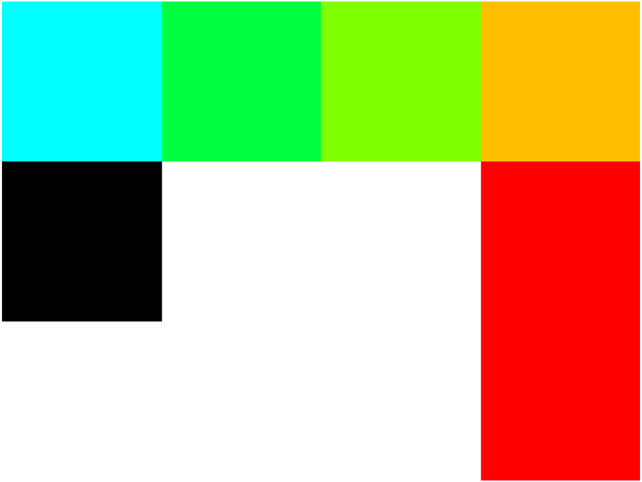

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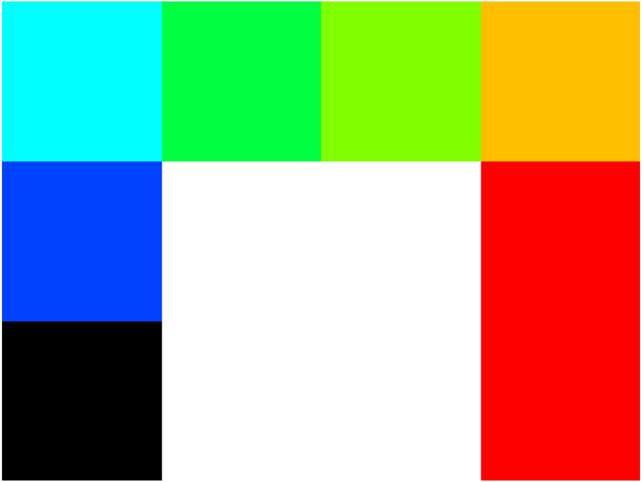

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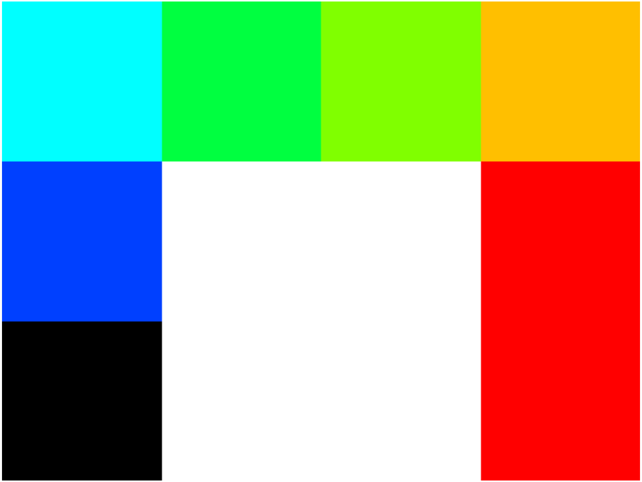

$$0=\rightarrow$$

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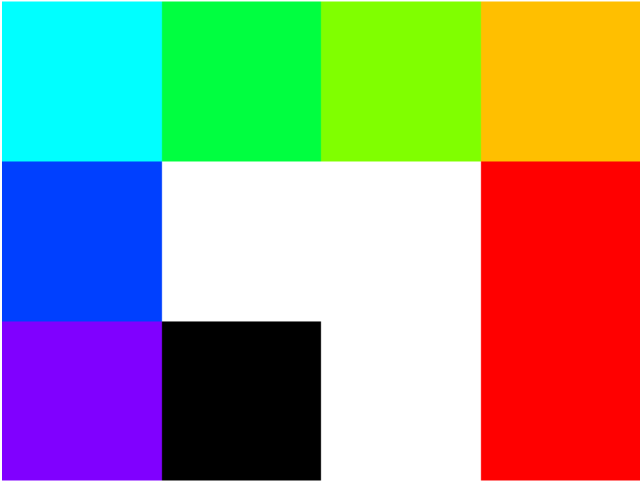

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What is a random walk (base 4)?

Pick a random number in $\{0,1,2,3\}$ and move $0 = \rightarrow$, $1 = \uparrow$, $2 = \leftarrow$, $3 = \downarrow$ ANIMATION

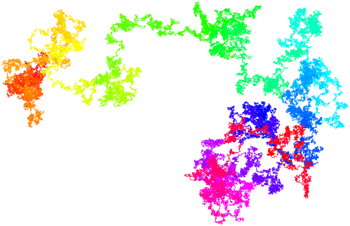

Figure : A million step base-4 pseudorandom walk. We use the spectrum to show when we visited each point (ROYGBIV and R).

Random walks look similarish

Chaos theory (order in disorder)

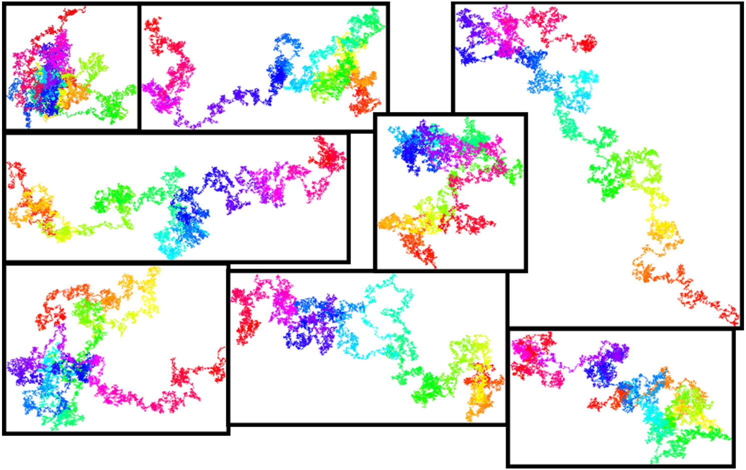

Figure : Eight different base-4 (pseudo)random[2] walks of one million steps.

[2] Python uses the *Mersenne Twister* as the core generator. It has a period of $2^{19937}-1 \approx 10^{6002}$.

Base-b random walks: Our direction choice

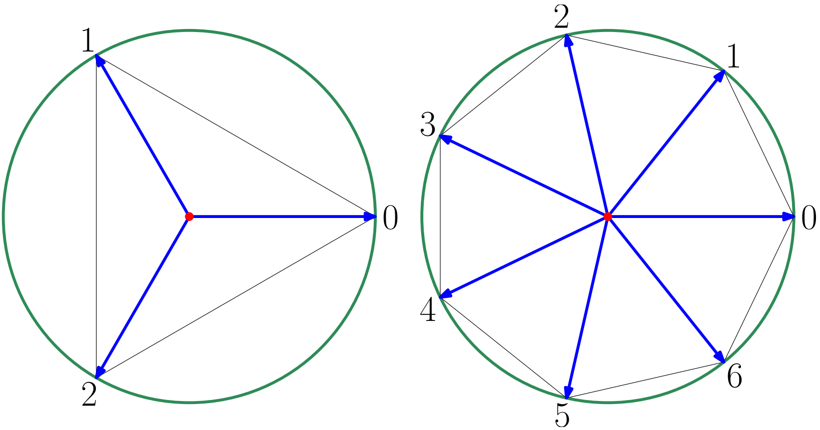

Figure : Directions for base-3 and base-7 random walks.

We are all base-four numbers (**AGCT**/U)

Contents

Two rational numbers

ANIMATION

The base-4 digit expansion of $Q1$ and $Q2$:

```
Q1=
0.221221012232121200122101223121001222100011232123121000122210001222
10001222100012221000012221000122201103010122010012010311033333333333
33333333333333330111111111111111111111111111110010000003003003200 32
00320030223000322203000322230003022220300032223000322230003222300032
22320000232223000322230032221330023321233023213232112112121222323233
33303000001000323003230032203032030110333011103301103101111011332333
3232322321221211211211223222222122...
```

```
Q2=
0.221221012232121200122101223121001222100011232123121000122210001222
10001222100012221000012221000122201103010122010012010311033333333333
33333333333333330111111111111111111111111111110010000003003003200 32
00320030223000322203000322230003022220300032223000322230003222300032
22320000232223000322230032221330023321233023213232112112121222323233
33303000001000323003230032203032030110333011103301103101111011000000
000000...
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Two rational numbers

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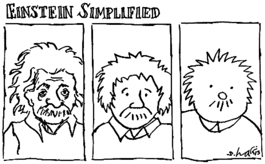

Figure : Self-referent walks on the rational numbers $Q1$ (top) and $Q2$ (bottom).

Two more rationals

Hard to tell from their decimal expansions

The following relatively small rational numbers [G. Marsaglia, 2010]

$$Q3 = \frac{3624360069}{7000000001} \quad \text{and} \quad Q4 = \frac{123456789012}{1000000000061},$$

have base-10 periods with huge length of **1,750,000,000** digits and **1,000,000,000,060** digits, respectively.

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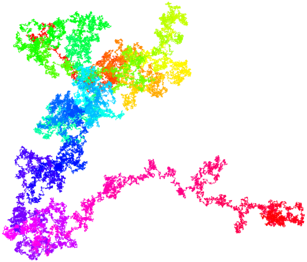
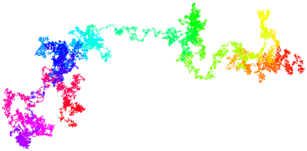


(a) $Q3$



(b) $Q4$

Figure : Walks on the first million base-10 digits of the rationals $Q3$ and $Q4$.

Walks on the digits of numbers

ANIMATION

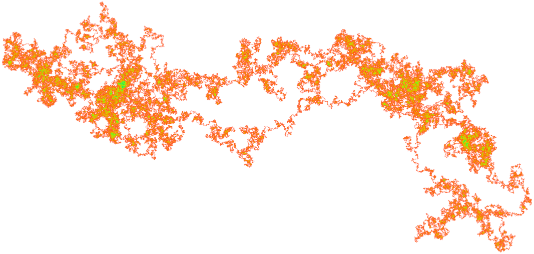


Figure : A walk on the first 10 million base-4 digits of π.

Walks on the digits of numbers

Coloured by hits (more pink is more hits)

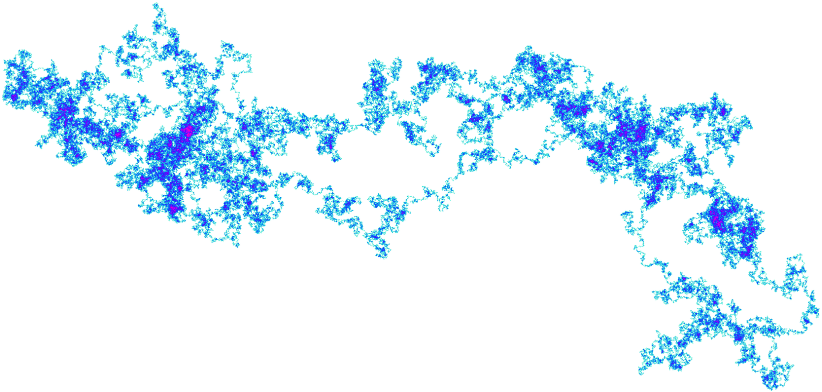


Figure : 100 million base-4 digits of π coloured by number of returns to points.

Contents

The Stoneham numbers $\alpha_{b,c} = \sum_{n=1}^{\infty} \frac{1}{c^n b^{c^n}}$

1973 Richard Stoneham proved some of the following (nearly 'natural') constants are b-normal for relatively prime integers b, c:

$$\alpha_{b,c} := \frac{1}{cb^c} + \frac{1}{c^2 b^{c^2}} + \frac{1}{c^3 b^{c^3}} + \ldots.$$

Such super-geometric sums are Stoneham constants. To 10 places

$$\alpha_{2,3} = \frac{1}{24} + \frac{1}{3608} + \frac{1}{3623878656} + \ldots$$

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Theorem (Nonnormality of Stoneham constants, Bailey–Borwein '12)

Given coprime $b \geq 2$ and $c \geq 2$, such that $c < b^{c-1}$, the constant $\alpha_{b,c}$ is bc-nonnormal.

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Such super-geometric sums are Stoneham constants. To 10 places

$$\alpha_{2,3} = \frac{1}{24} + \frac{1}{3608} + \frac{1}{3623878656} + \ldots$$

Theorem (Normality of Stoneham constants, Bailey–Crandall '02)

For every coprime pair of integers $b \geq 2$ and $c \geq 2$, the constant $\alpha_{b,c}$ is b-normal.

Theorem (Nonnormality of Stoneham constants, Bailey–Borwein '12)

Given coprime $b \geq 2$ and $c \geq 2$, such that $c < b^{c-1}$, the constant $\alpha_{b,c}$ is bc-nonnormal.

- Since $3 < 2^{3-1} = 4$, $\alpha_{2,3}$ is 2-normal and 6-nonnormal !

The Stoneham numbers $\alpha_{b,c} = \sum_{n=1}^{\infty} \frac{1}{c^n b^{c^n}}$

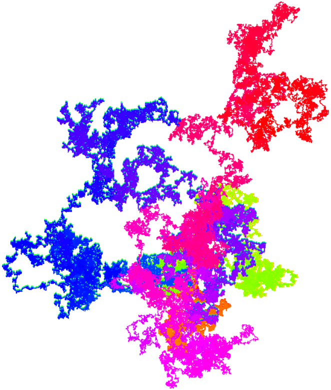

Figure : $\alpha_{2,3}$ is 2-normal (top) but 6-nonnormal (bottom). Is seeing believing?

The Stoneham numbers $\alpha_{b,c} = \sum_{n=1}^{\infty} \frac{1}{c^n b^{c^n}}$

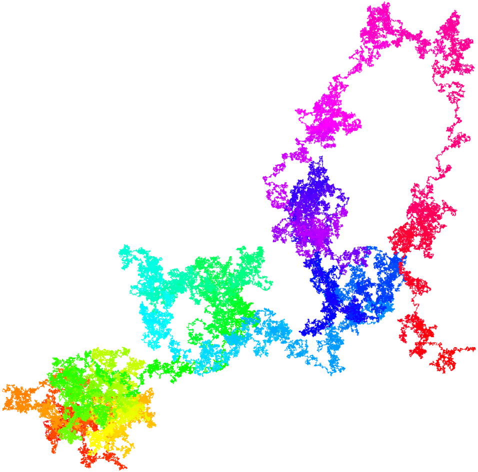

Figure : Is $\alpha_{2,3}$ 3-normal or not? Is it strongly 2-normal?

Contents

The expected distance to the origin $\frac{\sqrt{\pi N}}{2\,d_N} \to 1$

Theorem

The expected distance d_N to the origin of a base-b *random walk* of N steps behaves like to $\sqrt{\pi N}/2$.

The expected distance to the origin

$\frac{\sqrt{\pi N}}{2\,d_N} \to 1$

Theorem

The expected distance d_N to the origin of a base-b *random walk* of N steps behaves like to $\sqrt{\pi N}/2$.

Number	Base	Steps	Average normalized dist. to the origin: $\frac{1}{\text{Steps}}\sum_{N=2}^{\text{Steps}} \frac{\text{dist}_N}{\frac{\sqrt{\pi N}}{2}}$	Normal
Mean of 10,000 random walks	4	1,000,000	**1.00315**	Yes
Mean of 10,000 walks on the digits of π	4	1,000,000	**1.00083**	?
$\alpha_{2,3}$	3	1,000,000	0.89275	?
$\alpha_{2,3}$	4	1,000,000	0.25901	Yes
$\alpha_{2,3}$	6	1,000,000	108.02218	No
π	4	1,000,000	0.84366	?
π	6	1,000,000	0.96458	?
π	10	1,000,000	0.82167	?
π	10	1,000,000,000	0.59824	?
$\sqrt{2}$	4	1,000,000	0.72260	?
Champernowne C_{10}	10	1,000,000	59.91143	Yes

Contents

Number of points visited

For a 2D lattice

- The expected number of distinct points visited by an N-step random walk on a two-dimensional lattice behaves for large N like $\pi N/\log(N)$ (Dvoretzky–Erdős, **1951**).

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- For example, for $N = 10^6$ these bounds are $(199256.1, 203059.5)$, while $\pi N/\log(N) = 227396$, which overestimates the expectation.

Catalan's constant

$G = 1 + 1/4 + 1/9 + 1/16 + \cdots$

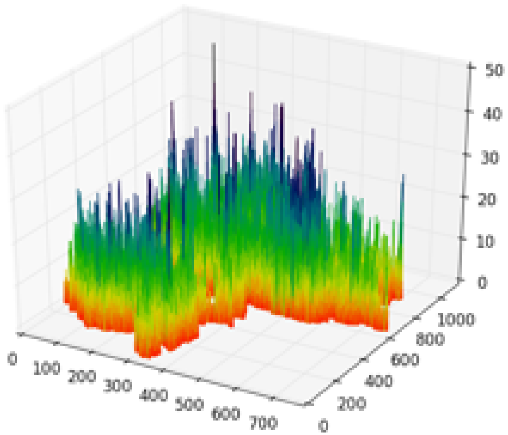
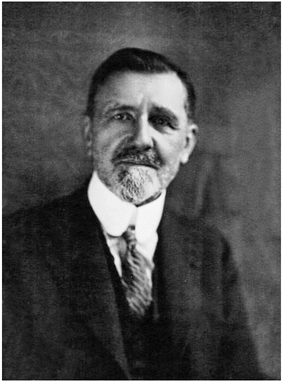


Figure : A walk on one million quad-bits of G with height showing frequency

Paul Erdős (1913-1996) "My brain is open"

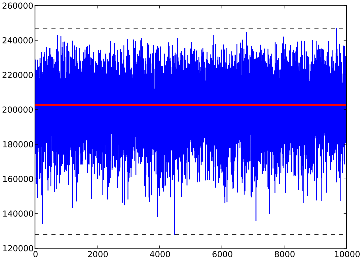
(a) Paul Erdős (Banff 1981. I was there)



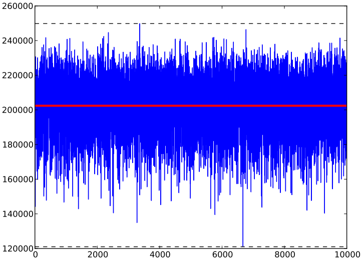
(b) Émile Borel (1871–1956)

Figure : Two of my favourites. Consult MacTutor.

Number of points visited: Again π looks random



(a) (Pseudo)random walks.



(b) Walks built by chopping up 10 billion digits of π.

Figure : Number of points visited by $10,000$ million-steps base-4 walks.

Points visited by various base-4 walks

Number	Steps	Sites visited	Bounds on the expectation of sites visited by a random walk	
			Lower bound	Upper bound
Mean of 10,000 random walks	1,000,000	**202,684**	199,256	203,060
Mean of 10,000 walks on the digits of π	1,000,000	**202,385**	199,256	203,060
$\alpha_{2,3}$	1,000,000	95,817	199,256	203,060
$\alpha_{3,2}$	1,000,000	195,585	199,256	203,060
π	1,000,000	**204,148**	199,256	203,060
π	10,000,000	1,933,903	1,738,645	1,767,533
π	100,000,000	16,109,429	15,421,296	15,648,132
π	1,000,000,000	138,107,050	138,552,612	140,380,926
e	1,000,000	176,350	199,256	203,060
$\sqrt{2}$	1,000,000	**200,733**	199,256	203,060
$\log 2$	1,000,000	**214,508**	199,256	203,060
Champernowne C_4	1,000,000	548,746	199,256	203,060
Rational number Q_1	1,000,000	378	199,256	203,060
Rational number Q_2	1,000,000	939,322	199,256	203,060

Normal numbers need not be so "random" ...

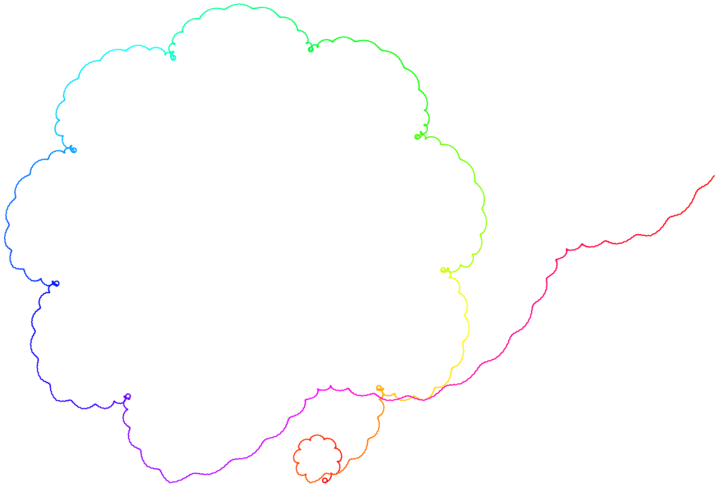


Figure : Champernowne $C_{10} = 0.123456789101112\ldots$ (normal).
Normalized distance to the origin: 15.9 (50,000 steps).

Normal numbers need not be so "random" ...

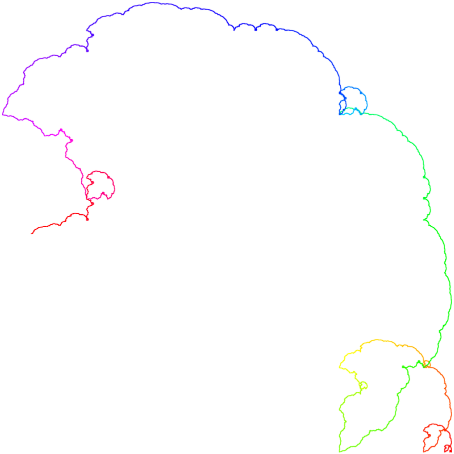


Figure : Champernowne $C_4 = 0.123101112132021\ldots$ (normal).
Normalized distance to the origin: 18.1 (100,000 steps).
Points visited: 52760. Expectation: (23333, 23857).

Normal numbers need not be so "random" ...

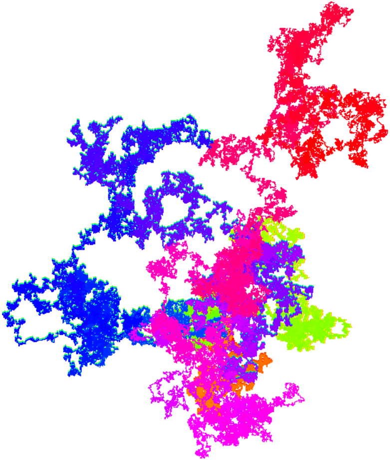


Figure : Stoneham $\alpha_{2,3} = 0.0022232032\ldots_4$ (normal base 4).
Normalized distance to the origin: 0.26 (1,000,000 steps).
Points visited: 95817. Expectation: (199256, 203060).

Normal numbers need not be so "random" ...

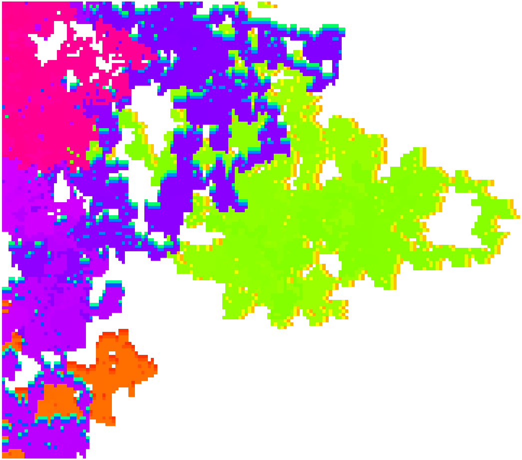


Figure : Stoneham $\alpha_{2,3} = 0.0022232032\ldots_4$ (normal base 4).
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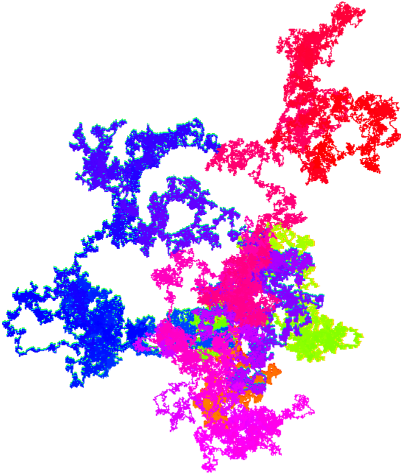
$\alpha_{2,3}$ is 4-normal but not so "random" ANIMATION



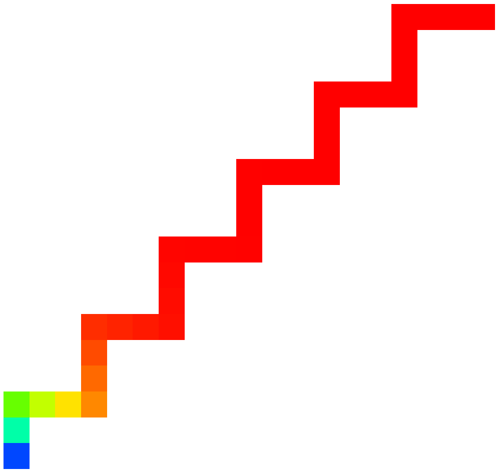


Figure : A pattern in the digits of $\alpha_{2,3}$ base 4. We show only positions of the walk after $\frac{3}{2}(3^n+1)$, $\frac{3}{2}(3^n+1)+3^n$ and $\frac{3}{2}(3^n+1)+2\cdot 3^n$ steps, $n=0,1,\ldots,11$.

Experimental conjecture

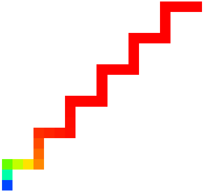
Proven 12-12-12 by Coons

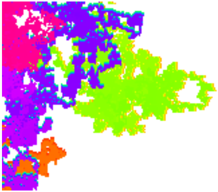
Theorem (Base-4 structure of Stoneham $\alpha_{2,3}$)

Denote by a_k the k^{th} digit of $\alpha_{2,3}$ in its base 4 expansion: $\alpha_{2,3} = \sum_{k=1}^{\infty} a_k/4^k$, with $a_k \in \{0,1,2,3\}$ for all k. Then, for all $n = 0,1,2,\ldots$ one has:

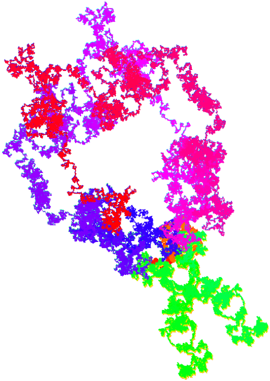
(i) $$\sum_{k=\frac{3}{2}(3^n+1)}^{\frac{3}{2}(3^n+1)+3^n} e^{a_k \pi i/2} = \begin{cases} -i, & \text{n odd} \\ -1, & \text{n even} \end{cases};$$

(ii) $a_k = a_{k+3^n} = a_{k+2\cdot 3^n}$ if $k = \frac{3(3^n+1)}{2}, \frac{3(3^n+1)}{2}+1, \ldots, \frac{3(3^n+1)}{2}+3^n-1$.





Likewise, $\alpha_{3,5}$ is 3-normal ... but not very "random" ANIMATION



Contents

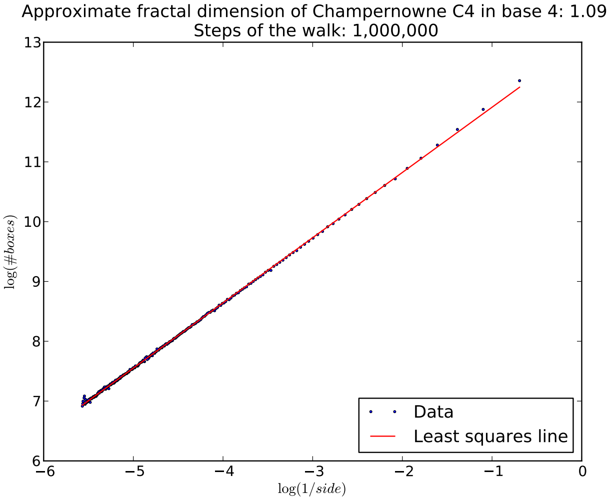
Box-dimension: Tends to ‘2’ for a planar random walk

$$\text{Box-dimension} = \lim_{\text{side}\to 0} \frac{\log(\#\ \text{boxes})}{\log(1/\text{side})}$$

Norway is “frillier” — *Hitchhiker’s Guide to the Galaxy*

Box-dimension:

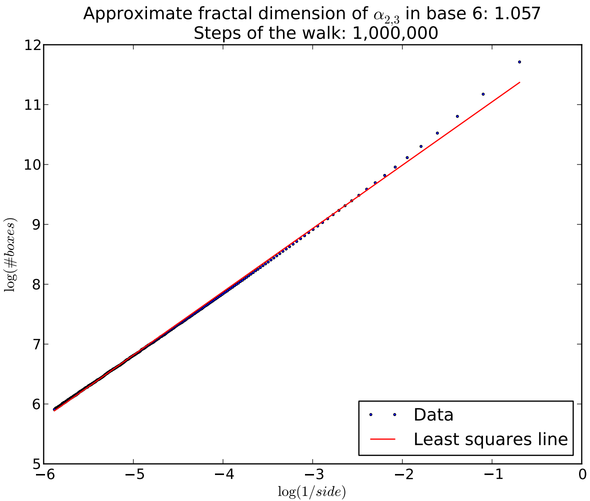
Tends to ‘2’ for a planar random walk



Fractals: self-similar (zoom invariant) partly space-filling shapes (clouds & ferns not buildings & cars). Curves have dimension 1, squares dimension 2

Box-dimension:

Tends to ‘2’ for a planar random walk



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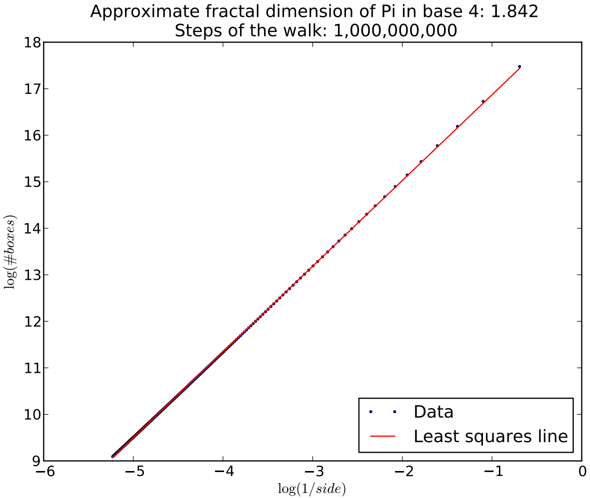
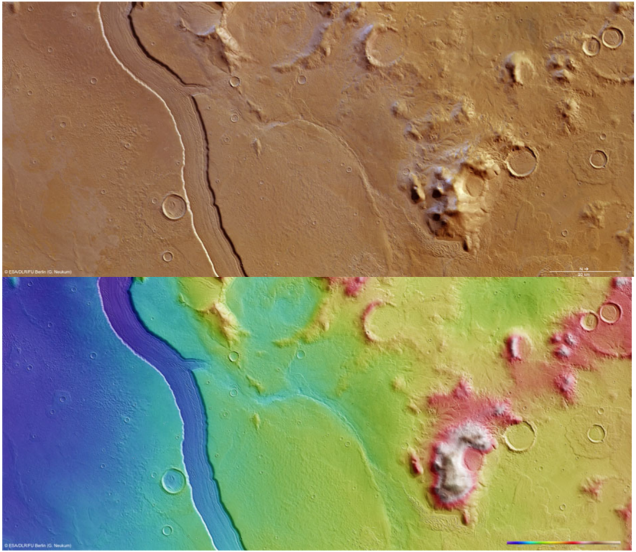


Fractals: self-similar (zoom invariant) partly space-filling shapes (clouds & ferns not buildings & cars). Curves have dimension 1, squares dimension 2

Contents

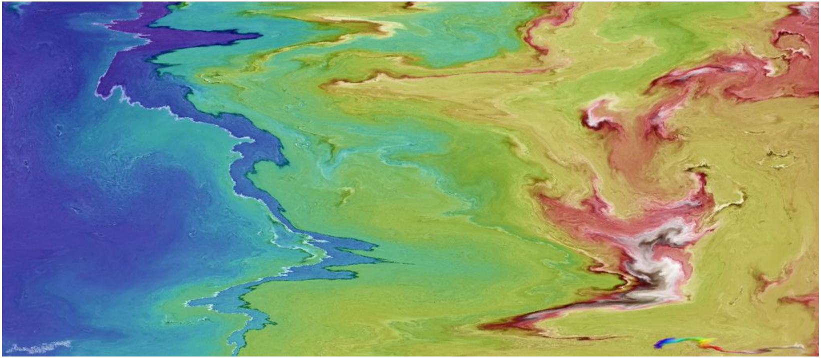
Fractals everywhere

From Mars



Fractals everywhere

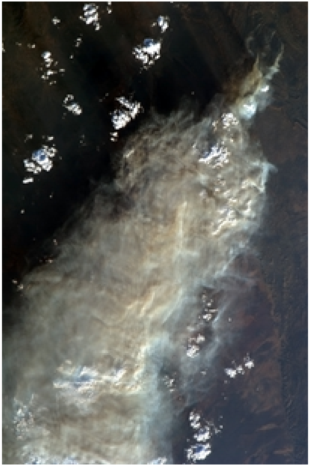
From Mars

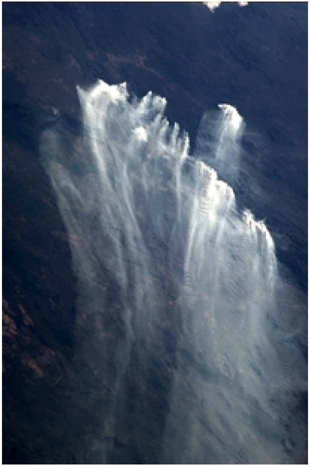


The picture fractalized by the Barnsley's
`http://frangostudio.com/frangocamera.html`

Fractals everywhere

From Space





Fractals everywhere

$1 \mapsto 3$ or $1 \mapsto 8$ or ...

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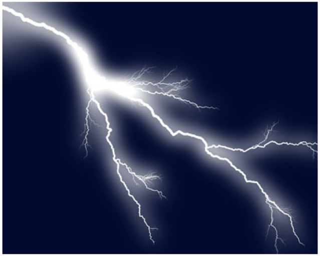

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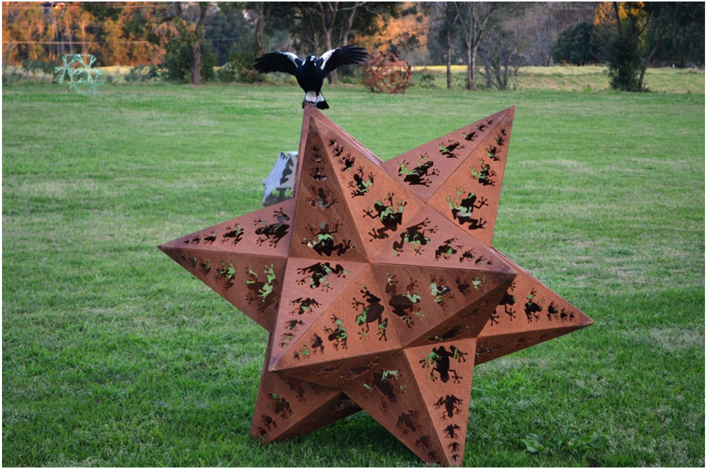


Fractals everywhere

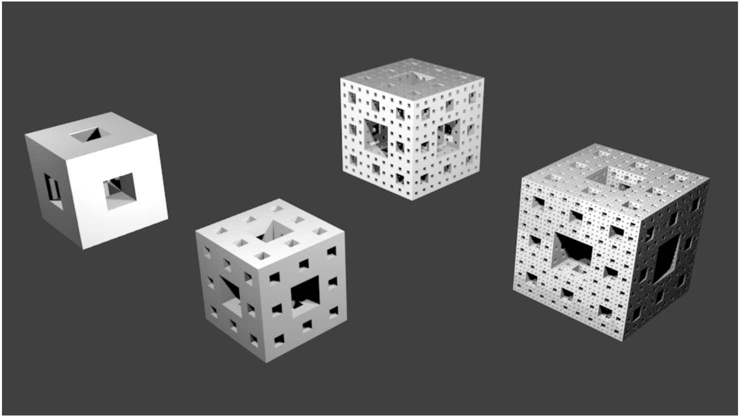
$1 \mapsto 3$ or $1 \mapsto 8$ or ...

Pascal triangle modulo two

[1] [1,1] [1,2,1] [1,3,3,1,] [1,4,6,4,1] [1,510,10,5,1] ...

Fractals everywhere

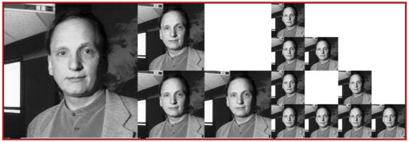
$1 \mapsto 3$ or $1 \mapsto 8$ or ...



Steps to construction of a Sierpinski cube

Fractals everywhere — The Sierpinski Triangle

$1 \mapsto 3 \mapsto 9$



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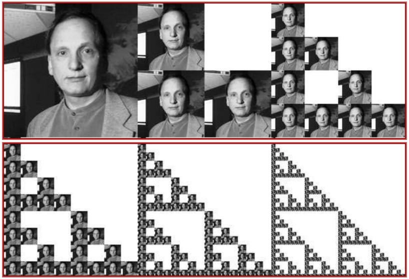

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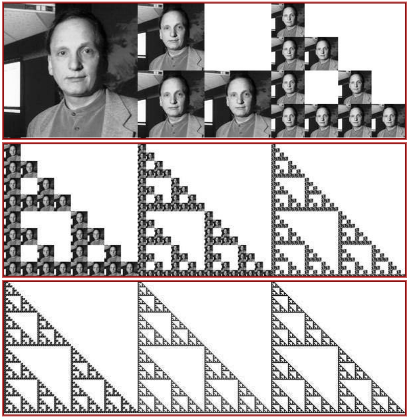

Fractals everywhere

The Sierpinski Triangle

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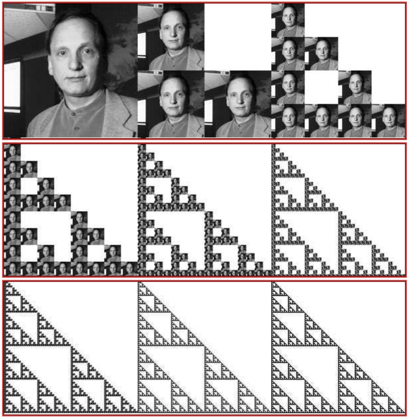
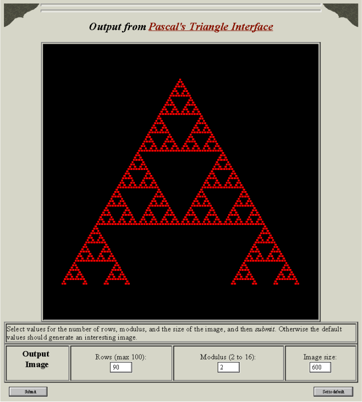




http://oldweb.cecm.sfu.ca/cgi-bin/organics/pascalform

Contents

Three dimensional walks: Using base six — soon on 3D screen

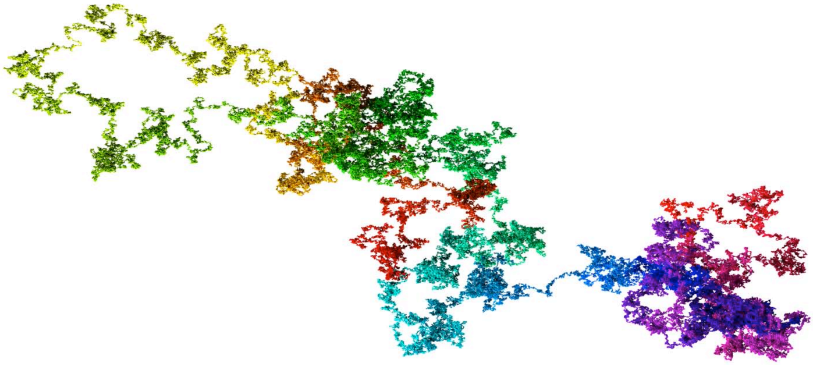


Figure : Matt Skerritt's 3D walk on π (base 6), showing one million steps. But 3D random walks are not recurrent.

Three dimensional walks: Using base six — soon on 3D screen

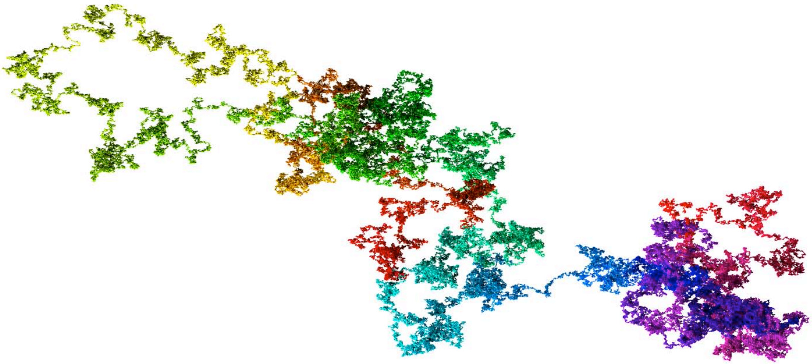
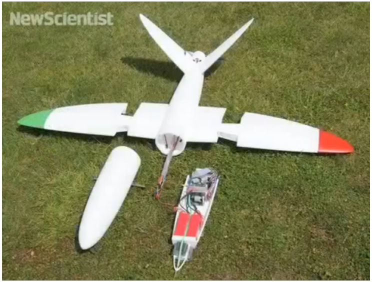


Figure : Matt Skerritt's 3D walk on π (base 6), showing one million steps. But 3D random walks are not recurrent.

"A drunken man will find his way home, a drunken bird will get lost forever." (Kakutani)

Three dimensional printing:

3D everywhere



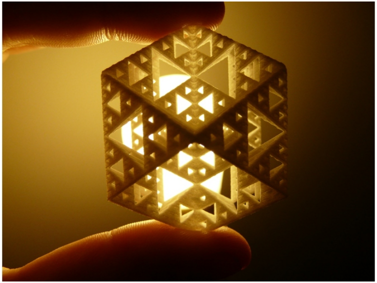


Figure : The future is here ...

www.digitaltrends.com/cool-tech/the-worlds-first-plane-created-entirely-by-3d-printing-takes-flight/

www.shapeways.com/shops/3Dfractals

Contents

Chaos games: Move half-way to a (random) corner

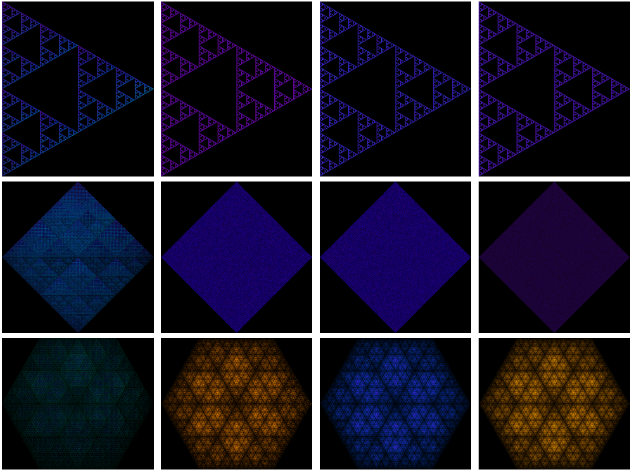


Figure : Coloured by frequency — leads to random fractals.
Row 1: Champernowne C_3, $\alpha_{3,5}$, random, $\alpha_{2,3}$. Row 2: Champernowne C_4, π, random, $\alpha_{2,3}$. Row 3: Champernowne C_6, $\alpha_{3,2}$, random, $\alpha_{2,3}$.

Contents

Automatic numbers: Thue–Morse and Paper-folding

Automatic numbers are never normal. They are given by simple but fascinating rules...giving structured/boring walks:

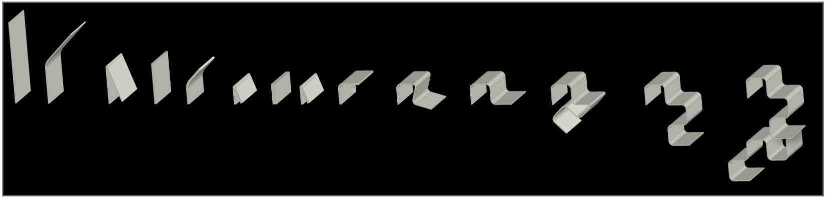


Figure : **Paper folding**. The sequence of left and right folds along a strip of paper that is folded repeatedly in half in the same direction. Unfold and read 'right' as '1' and 'left' as '0': 1 0 1 1 0 0 1 1 1 0 0 1 0 0

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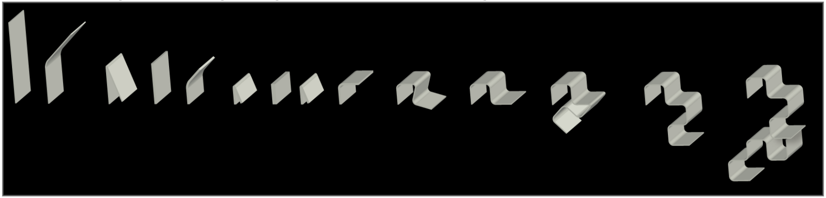

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Thue–Morse constant (transcendental; 2-automatic, hence nonnormal):

$$TM_2 = \sum_{n=1}^{\infty} \frac{1}{2^{t(n)}} \text{ where } t(0) = 0, \text{ while } t(2n) = t(n) \text{ and } t(2n+1) = 1 - t(n)$$

$$0.01101001100101101001011001101001\ldots$$

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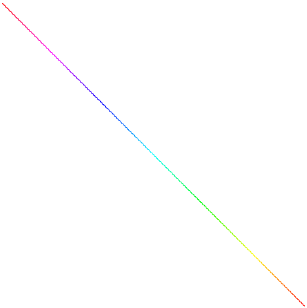
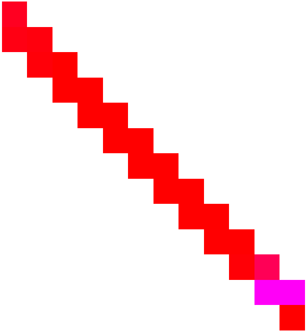


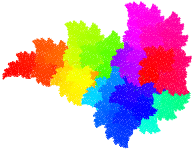
(a) 1,000 bits of Thue–Morse sequence.



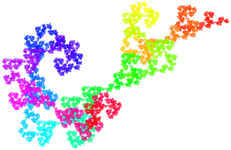
(b) 10 million bits of paper-folding sequence.

Figure : Walks on two automatic and so nonnormal numbers.

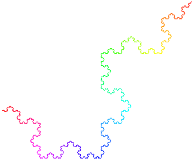
Automatic numbers: Turtle plots look great!



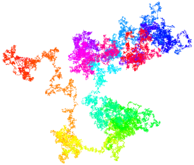
(a) Ten million digits of the paper-folding sequence, rotating 60°.



(b) One million digits of the paper-folding sequence, rotating 120° (a dragon curve).



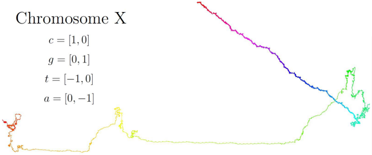
(c) 100,000 digits of the Thue–Morse sequence, rotating 60° (a Koch snowflake).

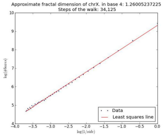


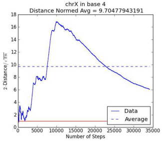
(d) One million digits of π, rotating 60°.

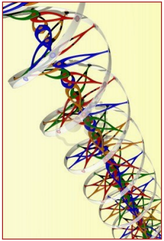
Figure : Turtle plots on various constants with different rotating angles in base 2—where ‘**0**’ yields forward motion and ‘**1**’ rotation by a fixed angle.

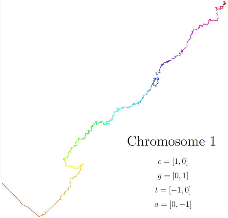
Genomes as walks: we are all base 4 numbers (ACGT/U)

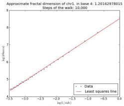


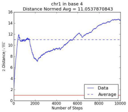




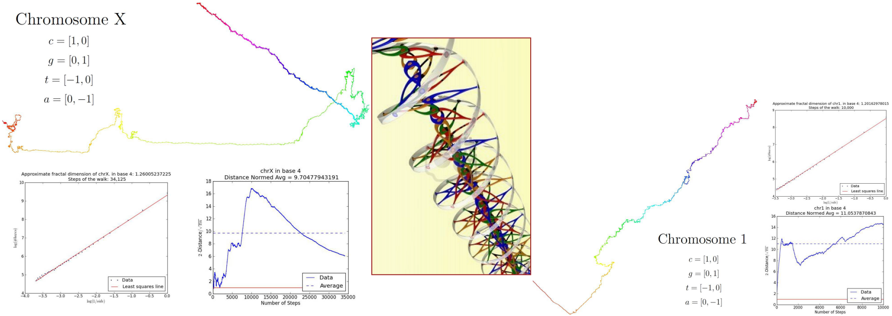






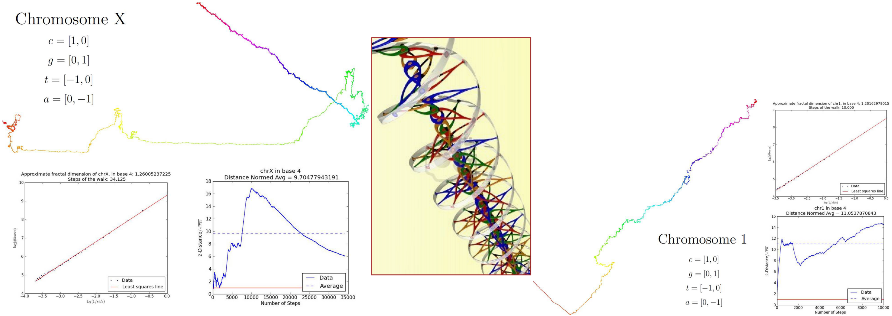


Genomes as walks: we are all base 4 numbers (ACGT/U)



The X Chromosome (34K) and Chromosome One (10K).

Genomes as walks: we are all base 4 numbers (ACGT/U)



The X Chromosome (34K) and Chromosome One (10K).

Ⓡ Chromosomes look less like π and more like concatenation numbers?

DNA for Storage: we are all base 4 numbers (ACGT/U)

News › Science › Biochemistry and molecular biology

Shakespeare and Martin Luther King demonstrate potential of DNA storage

All 154 Shakespeare sonnets have been spelled out in DNA to demonstrate the vast potential of genetic data storage

Ian Sample, science correspondent
The Guardian, Thursday 24 January 2013
Jump to comments (...)

When written in DNA, one of Shakespeare's sonnets weighs 0.3 millionths of a millionth of a gram. Photograph: Oli Scarff/Getty

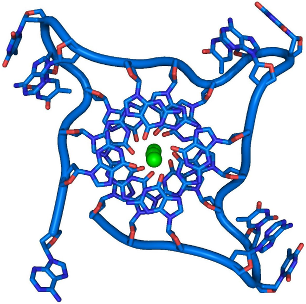


Figure : The potential for DNA storage (L) and the quadruple helix (R)

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2012 walk on π (went *viral*)

Biggest mathematics picture ever?

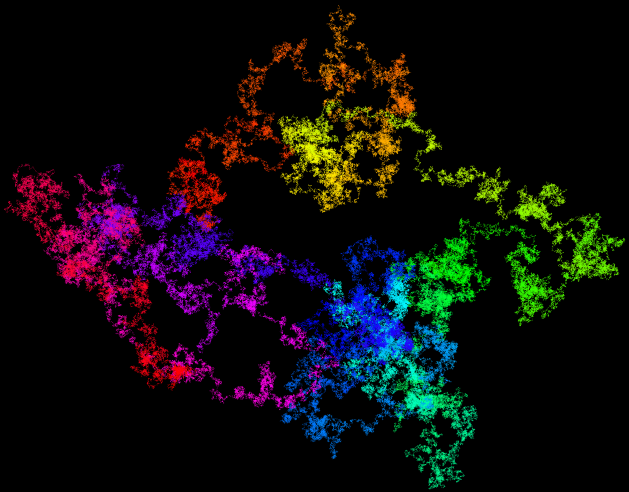

Figure : Walk on first 100 billion base-4 digits of π (normal?).

2012 walk on π (went *viral*)

Biggest mathematics picture ever?

Resolution: 372,224×290,218 pixels
(108 gigapixels)

Computation: took roughly a month where several parts of the algorithm were run in parallel with 20 threads on CARMA's MacPro cluster.

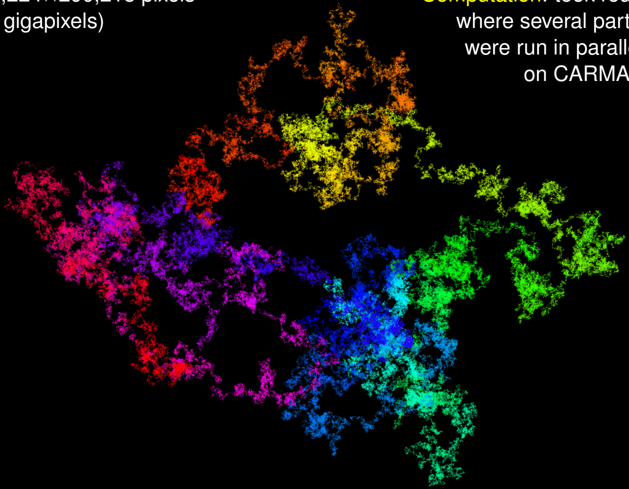

Figure : Walk on first 100 billion base-4 digits of π (normal?).

`http://gigapan.org/gigapans/106803`

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The Aperiodical

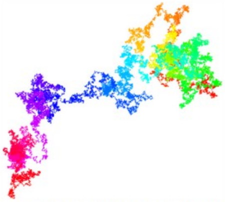
Share some maths | About the Aperiodical | Carnival of Mathematics | Seen some good new research? | Search

WLTM real number. Must be normal and enjoy long walks on the plane

By Christian Perfect On June 7, 2012 · 1 Comment · In News, Uncategorized

Something that whipped round Twitter over the weekend is an early version of a paper by Francisco Aragón Artacho, David Bailey, Jonathan Borwein and Peter Borwein, investigating the usefulness of planar walks on the digits of real numbers as a way of measuring their randomness.

A problem with real numbers is to decide whether their digits (in whatever base) are "random" or not. As always, a strict definition of randomness is up to either the individual or the enlightened metaphysicist, but one definition of randomness is normality – every finite string of digits occurs with uniform asymptotic frequency in the decimal (or octal or whatever) representation of the number. Not many results on this subject exist, so people try visual tools to see what randomness looks like, comparing potentially normal numbers like π with pseudorandom and non-random numbers. In fact, the (very old) question of whether π is normal was one of the main motivators for this study.



A million step walk on the concatenation of the base 10 digits of the prime numbers, converted to base 4

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http://aperiodical.com/2012/06/wltm-real-number-must-be-normal-and-enjoy-long-walks-on-the-plane/

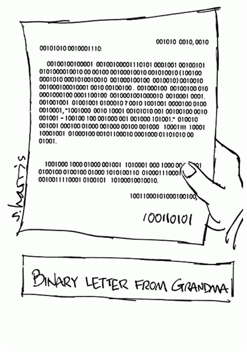

Figure : Is Grandma's letter normal?


SOCIAL DIMENSION

A Random Walk with Pi

By Samuel Arbesman | June 12, 2012 | 12:12 pm | Categories: Science Blogs, Social Dimension

Photo: Myki Roventine/Flickr

We've gotten really good at generating big datasets. From what we search for on Google to all the stuff we do on Facebook, we generate a lot of data. And there have in turn been a proliferation of methods of visualizing big data, slicing and dicing it and providing meaning to it.

But what's the mother of all big datasets? The digits of π. Of course, the number of digits in π is infinite, but over the years, as computers have become more powerful, we know many more of them than ever before.

But it's one thing to say that I can download trillions of digits of π and another to make some sort of sense out of all of these digits. Into this stage a team of researchers from Australia, Lawrence Berkeley National Laboratory, and Simon Fraser University with some elegant visualizations of π. In a recent paper, they used a classic method of visualizing large strings of numbers: the random walk.

A true random walk is the path described by a sequence of randomly generated numbers. For example, if we have a string of -1's and 1's, then one way to describe this is by using time as the x-axis, and the y-axis as the cumulative sum of numbers so far. This yields a nice little sawtooth shape.

Alternatively, you can do a random walk on a plane, with each digit describing the direction and distance being moved. In the case of π and the method used by these researchers, they used color to designate additional information. For example, here color is used to designate time, with the blue being earlier in the sequence and the further along the spectrum is later in the sequence.

http://www.wired.com/wiredscience/2012/06/a-random-walk-with-pi/

Especially in Japan

Figure : Decisions, decisions

http://wired.jp/2012/06/15/a-random-walk-with-pi/


wired.jp/category/Science/
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2 日常世界のラブドー
LATEST NEWS
円周率をランダムウォークで視覚化
国際的な研究者チームが、ランダムウォークというモデルを使って、1,000億桁に及ぶ円周率を視覚化した。ほかの定数の視覚化もあり、円周率が非常に「ランダム」であることがよくわかる。
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WIREDの未来生物学講義
ワイアード VOL.4 5月10日 発売
wired.jp/2012/06/15/a-random-walk-with-pi/

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円周率をランダムウォークで視覚化
2. 186 RT
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グーグルマップと別れ、アップルは成功への道を走れるか
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5. 「新MacBook Proは種の進化だ」

HOTTEST TOPIC

1. 276 RT Random walk in the Pi visualization
2. 186 RT The new MacBook Pro is "almost impossible to repair"
3. 90 RT Apple and Google are, kill a dedicated terminal at GPS
4. 85 RT Google Maps and separation, or Apple can run on the road to success
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RANKING

1. Random walk in the Pi visualization
2. The new MacBook Pro is "almost impossible to repair"
3. Apple and Google are, kill a dedicated terminal at GPS
4. Google Maps and separation, or Apple can run on the road to success
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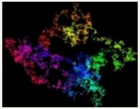
Illustration



Walking on pi

By **Francisco Javier Aragón Artacho** · Sep 21, 2012

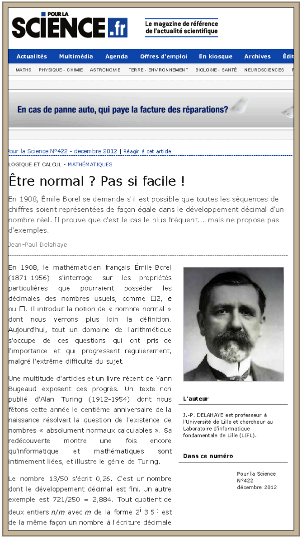
6 Comments

180 votes

Learn about Pi at http://www.carma.newcastle.edu.au/jon/pi-2012.pdf

October 25 2012: Music and Maths Concert http://carma.newcastle.edu.au/pdf/music_maths.pdf

Hear Pi at http://carma.newcastle.edu.au/walks/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En cas de panne auto, qui paye la facture des réparations?

Pour la Science N°422 - decembre 2012 | Réagir à cet article

LOGIQUE ET CALCUL - MATHÉMATIQUES

Être normal ? Pas si facile !

En 1908, Émile Borel se demande s'il est possible que toutes les séquences de chiffres soient représentées de façon égale dans le développement décimal d'un nombre réel. Il prouve que c'est le cas le plus fréquent... mais ne propose pas d'exemples.

Jean-Paul Delahaye

En 1908, le mathématicien français Émile Borel (1871-1956) s'interroge sur les propriétés particulières que pourraient posséder les décimales des nombres usuels, comme □2, e ou □. Il introduit la notion de « nombre normal » dont nous verrons plus loin la définition. Aujourd'hui, tout un domaine de l'arithmétique s'occupe de ces questions qui ont pris de l'importance et qui progressent régulièrement, malgré l'extrême difficulté du sujet.

Une multitude d'articles et un livre récent de Yann Bugeaud exposent ces progrès. Un texte non publié d'Alan Turing (1912-1954) dont nous fêtons cette année le centième anniversaire de la naissance résolvait la question de l'existence de nombres « absolument normaux calculables ». Sa redécouverte montre une fois encore qu'informatique et mathématiques sont intimement liées, et illustre le génie de Turing.

Le nombre 13/50 s'écrit 0,26. C'est un nombre dont le développement décimal est fini. Un autre exemple est 721/250 = 2,884. Tout quotient de deux entiers n/m avec m de la forme $2^i\ 3\ 5^j$ est de la même façon un nombre à l'écriture décimale

L'auteur

J.-P. DELAHAYE est professeur à l'Université de Lille et chercheur au Laboratoire d'informatique fondamentale de Lille (LIFL).

Dans ce numéro

Pour la Science N°422 décembre 2012

December 2012: Normality of Pi and Stoneham numbers

Our analysis of 5 trillion hex-digits suggests π is *very* probably normal!

http://www.pourlascience.fr/ewb_pages/f/fiche-article-etre-normal-pas-si-facile-30713.php

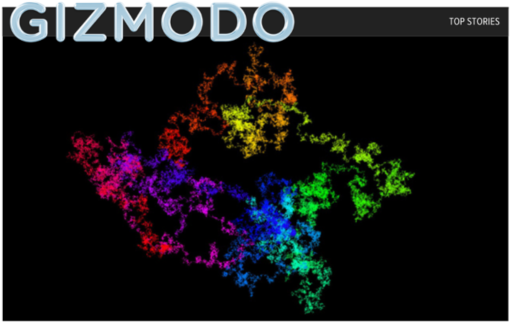


IMAGE CACHE

What Is This?

Jamie Condliffe

JAN 10, 2013 7:45 AM 21,386 102

Share +1 Like 152

This ragged cloud of color looks messy and unstructured—but in fact it's a rare and unusual view of one of the most fundamental things in science. Can you work out what it is?

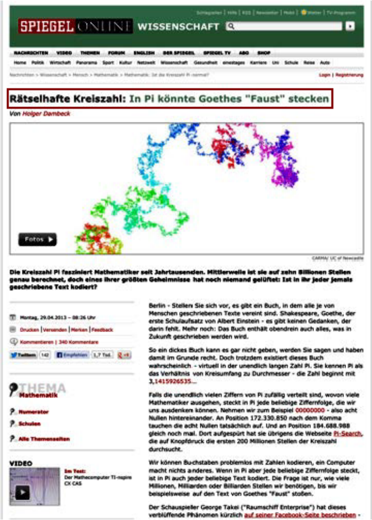
Sadly for you, we're going to let you puzzle over the answer for a little while. To stop you all going round in circles, though, here are a couple of clues: it was generated by a computer and the thing it depicts is used in every branch of science, from mathematics to engineering.

We'll post the solution here in an hour or so. Until then, try and work out exactly what it is amongst yourselves in the comments—without cheating and resorting to Google Images.

Update: You can find the answer here.

January 10, 2013 `http://gizmodo.com/5974779/what-is-this`

- Spiegel. The mysterious circular number: Pi contains Goethe
  (not Shakespeare)



SPIEGEL ONLINE WISSENSCHAFT

Rätselhafte Kreiszahl: In Pi könnte Goethes "Faust" stecken

Von Holger Dambeck

CARMA/ UC of Newcastle

Die Kreiszahl Pi fasziniert Mathematiker seit Jahrtausenden. Mittlerweile ist sie auf zehn Billionen Stellen genau berechnet, doch eines ihrer größten Geheimnisse hat noch niemand gelüftet: Ist in ihr jeder jemals geschriebene Text kodiert?

Montag, 29.04.2013 – 08:26 Uhr

Berlin - Stellen Sie sich vor, es gibt ein Buch, in dem alle je von Menschen geschriebenen Texte vereint sind. Shakespeare, Goethe, der erste Schulaufsatz von Albert Einstein - es gibt keinen Gedanken, der darin fehlt. Mehr noch: Das Buch enthält obendrein auch alles, was in Zukunft geschrieben werden wird.

So ein dickes Buch kann es gar nicht geben, werden Sie sagen und haben damit im Grunde recht. Doch trotzdem existiert dieses Buch wahrscheinlich - virtuell in der unendlich langen Zahl Pi. Sie kennen Pi als das Verhältnis von Kreisumfang zu Durchmesser - die Zahl beginnt mit 3,1415926535...

Falls die unendlich vielen Ziffern von Pi zufällig verteilt sind, wovon viele Mathematiker ausgehen, steckt in Pi jede beliebige Ziffernfolge, die wir uns ausdenken können. Nehmen wir zum Beispiel 00000000 - also acht Nullen hintereinander. An Position 172.330.850 nach dem Komma tauchen die acht Nullen tatsächlich auf. Und an Position 184.688.988 gleich noch mal. Dort aufgespürt hat sie übrigens die Webseite Pi-Search, die auf Knopfdruck die ersten 200 Millionen Stellen der Kreiszahl durchsucht.

Wir können Buchstaben problemlos mit Zahlen kodieren, ein Computer macht nichts anderes. Wenn in Pi aber jede beliebige Ziffernfolge steckt, ist in Pi auch jeder beliebige Text kodiert. Die Frage ist nur, wie viele Millionen, Milliarden oder Billiarden Stellen wir benötigen, bis wir beispielsweise auf den Text von Goethes "Faust" stoßen.

Der Schauspieler George Takei ("Raumschiff Enterprise") hat dieses verblüffende Phänomen kürzlich auf seiner Facebook-Seite beschrieben -

April 29, 2013 www.spiegel.de/wissenschaft/mensch/mathematik-ist-die-kreiszahl-pi-normal-a-895876.html

- **Guardian**. 3.14.14

theguardian

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News › Science › Alex Bellos's Adventures in Numberland

ALEX'S ADVENTURES IN NUMBERLAND
HOSTED BY THE GUARDIAN

Previous Blog home Next

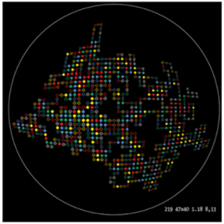
Pi Day: pi transformed into incredible art – in pictures

The number 3.14159 ... as you have never seen it before. Striking computer-generated images of the most famous number in maths

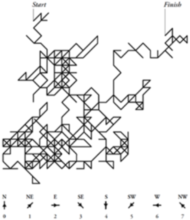
Pi Day: Shakespeare, Jane Austen and the poet laureate of pi

Share g+1 219 Share 42 Email

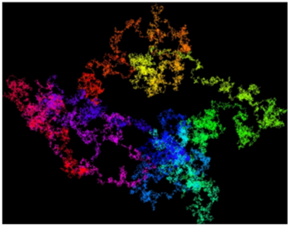
Alex Bellos
theguardian.com, Friday 14 March 2014 18.05 AEST



Protein fix: Martin Krzywinski, a well known pi artist, here lets the first 768 decimal digits of pi behave like the string of amino acids in a protein. In other words, he arbitrarily lets the prime digits (2,3,5 and 7) be black dots. The remaining dots each are colour-coded. A computer algorithm then 'folds' the string in such a way to maximize the number of adjacent black dots. He said: "The colour scheme is inspired by the Bauhaus movement. By smoothly deforming the lattice that holds the path into a circle and adjusting the sizes of digit circles, we can get something that starts to look like a



The first person to visualize the random nature of pi's decimal digits was the Victorian mathematician John Venn. In *The Logic of Chance* (1888), he suggested that the digits 0 to 7 represent eight compass directions, and he followed the path tracked by these digits in pi. He misses out the initial 3, and starts 14159. Venn's image was the first "random walk", an idea now used frequently in probability and statistics. (The illustration is taken from my book, Alex's Adventures in Numberland)



Francisco Aragón and his colleages converted pi into base 4, meaning that it is written using only the digits 0, 1, 2 and 3, and with these digits representing north, south, east and west, tracked a

March 14, 2014 www.theguardian.com/science/alexs-adventures-in-numberland/gallery/2014/mar/14/pi-day-pi-transformed-into-incredible-art-in-pictures

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http://carma.newcastle.edu.au/walks/

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