BULL. AUSTRAL. MATH. SOC. 10E05
VOL. 14 (1976), 397-398.

An addition to a note of mine

Kurt Mahler

A recent improvement by H. Guggenheimer of the lower bound for the product of the volumes of a pair of polar reciprocal convex bodies is used to replace one of the results of Kurt Mahler, "Polar analogues of two theorems by Minkowski", *Bull. Austral. Math. Soc.* 11 (1974), 121-129 by a best possible one. A similar improvement can be made for the other theorems in my note.

In my note [2], I established three theorems on the connection between symmetric convex bodies in R^n and hyperplane lattices in R^n. The proofs of these theorems were based on the inequality

$$V(K)V(K^*) \geq n^{-n/2}\pi^n\Gamma\left((n/2)+1\right)^{-2}$$

for the volumes of a pair of polar reciprocal convex bodies K and K^*.

Recently, Guggenheimer [1] has replaced this inequality by

$$V(K)V(K^*) \geq 4^n/n! ,$$

which is best possible; for equality holds if one of the two bodies K, K^* is an n-dimensional parallelepiped.

On using this improved lower bound in the proofs of my note, but without further changes, my first theorem takes now the following simple form.

THEOREM 1. *Let* $K : F(\mathrm{x}) \leq 1$ *be a symmetric convex body of volume*

$$V(K) \geq 2^n/n! .$$

Then there exists an integral vector $u \neq 0$ *such that* $F(\mathrm{x}) \geq 1$ *at all*

Received 11 February 1976.

real points $\mathbf{x}$ *satisfying* $\mathbf{u}.\mathbf{x} = 1$.

This theorem is best possible as is immediately seen when K is the octahedron

$$|x_1| + \ldots + |x_n| \le 1 ,$$

and $\mathbf{u}$ is the vector $(1, 0, \ldots, 0)$.

Theorem 1 implies the following analogue to Minkowski's Theorem on linear forms.

Let

$$y_h = a_{h1}x_1 + \ldots + a_{hn}x_n \quad (h = 1, 2, \ldots, n)$$

be n *linear forms with real coefficients of determinant* 1 . *Then there exist integers* $u_1, \ldots, u_n$ *not all zero such that* $|y_1| + \ldots + |y_n| \ge 1$ *for all real* $x_1, \ldots, x_n$ *satisfying* $u_1x_1 + \ldots + u_nx_n = 1$.

The same change allows one to improve the other two theorems of my note.

References

[1] H. Guggenheimer, "Polar reciprocal convex bodies", *Israel J. Math.* **14** (1973), 309-316.

[2] Kurt Mahler, "Polar analogues of two theorems by Minkowski", *Bull. Austral. Math. Soc.* **11** (1974), 121-129.

Department of Mathematics,
Institute of Advanced Studies,
Australian National University,
Canberra, ACT.